Optimal Monetary Responses to News of an Oil Discovery

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Abstract

How should monetary policy respond to an oil discovery? Oil discoveries provide news that natural output will rise in the future, which lowers the natural real interest rate. Optimal policy must accommodate these changes, and is well-approximated by a Taylor rule that includes the natural real rate. Ignoring these changes, as in currency pegs or naive Taylor rules, causes forward-looking inflation and a recession. I prove this by analytically deriving optimal policy in a small open economy with oil and news shocks, and then discuss the results using stylised illustrations.

Keywords: News shock, oil, optimal monetary policy, small open economy.

JEL Classification: E52, E62, F41, O13, Q30, Q33

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1 Introduction

An oil discovery provides news about the future. In the months and years that follow contracts will be negotiated, wells will be drilled, and after some delay oil revenues will feed into the domestic economy, typically based on the spending choices of the government (Pieschacon, 2012; Arezki et al., 2017). During this delay households and firms can react to the knowledge of the impending windfall, causing changes in the economy. However, because these changes are driven by news, rather than contemporaneous shocks, it is unclear how the central bank should respond. This paper aims to fill that gap.

The monetary response to an oil discovery matters because, of the approximately 200 sovereign states in the world, 130 are endowed with natural resources and 47 are resource-dependent (by the IMF's definition; Baunsgaard et al., 2012). Recent major oil and gas discoveries have been made in the US, UK and Australia amongst others. I will show that the majority of resource-dependent countries are conducting monetary policy poorly, as three-quarters follow some form of exchange rate target (split evenly between developed and developing countries; IMF, 2016). More broadly, oil discoveries are a rare example of a large, well-identified news shock (Arezki et al., 2017), and so studying how monetary policy should respond may yield insights relevant for news shocks more generally.

To study this I extend a standard New Keynesian small open economy model (Gali and Monacelli, 2005; 2008) in two ways: by including a government that receives oil revenues, and allowing households to respond to news. I assume that households consume both home and foreign goods, have perfect access to international borrowing, and their wealth increases relative to their foreign counterparts when oil is discovered. Firms set prices according to Calvo (1983). Oil is produced using both reserves and capital, which takes time to build, and the revenues accrue to the government which spends them according to simple fiscal rules. The model is sufficiently parsimonious that it allows me to analytically derive both the central bank's objective function and optimal policy. To illustrate that optimal policy matters, I compare it to commonly used Taylor rules and an exchange rate peg under a variety of fiscal rules, illustrated in stylized examples.

I find that an oil discovery lowers the natural real interest rate until oil production peaks, and optimal monetary policy should do the same with the policy rate. The natural real rate falls because households and firms expect output to be higher in the future. Firms therefore raise prices in anticipation, which suppresses consumption. Optimal monetary policy offsets this by cutting rates to boost consumption and close the output gap, and this is well-approximated by a Taylor rule that includes the natural interest rate.

If monetary policy fails to move with the natural interest rate, for example under widely-used exchange rate pegs or naive Taylor rules, it can cause a recession. The reason is because of the forward-looking behaviour of households and firms. Consider an exchange rate peg, as used by 75% of resource-dependent economies. When oil is discovered the real exchange rate will need to appreciate twice: immediately as households learn they are wealthier and consume more, and at production as the government spends the oil revenues. If the nominal exchange rate is fixed then the real exchange rate can only appreciate through higher domestic prices, which are sticky. Between discovery and production inflation will therefore be both backward-looking - as firms 'catch-up' on the first appreciation, and forward-looking - as firms anticipate the second appreciation. The latter depresses output below its natural level, creating a negative output gap and stagflation. A naive Taylor rule will respond aggressively to forward-looking inflation, exacerbating the negative output gap further. This can be avoided if the rule incorporates changes to the natural interest rate.

This work contributes to three main strands of literature. The first is on monetary policy in oil exporters. The literature on oil and monetary policy has focused on oil importers (Hamilton, 1983; Bernanke et al., 1997; Elekdag et al., 2007; Blanchard and Gali, 2007; Kilian, 2009; Nakov and Pescatori 2010; Kilian and Lewis, 2011; Bodenstein et al., 2012; Dagher et al., 2010). For oil exporters there has been a considerable body of work on medium to long term fiscal policy (see survey by van der Ploeg and Venables, 2012; Pieschacon, 2012; van den Bremer et al., 2016). However, the majority of work on monetary policy in oil exporters was done without the benefit of DSGE models. For example, Eastwood and Venables (1982) were motivated by the UK's North Sea oil and found that an oil discovery immediately causes the exchange rate to appreciate due to raised expectations, which can cause a recession if not immediately offset by increased demand (see also Buiter and Purvis, 1983; van Wijnbergen, 1984; Neary and van Wijnbergen, 1984). More recent work has started to incorporate oil exports into modern New Keynesian frameworks (Catao and Chang, 2013; Bergholt, 2014). This literature has focused on oil price shocks, such as Ferrero and Seneca (2015) who find that optimal policy should cut when the oil price falls if the economy has substantial spillovers from the oil sector; and Gross and Hansen (2018) who find that taxation is more efficient than monetary policy at stabilizing commodity price shocks, within a class of numerically-optimized Taylor rules. This is the first paper to study how monetary policy should optimally respond to an oil discovery.

Second, this paper also contributes to the growing literature on news shocks, and provides the first study of how monetary policy should optimally respond to news about future demand. The news shock literature has focused on Pigou (1927) cycles, where good news about future productivity (rather than demand as studied in this paper) generates positive co-movement in consumption, investment and hours worked today (see review by Lorenzoni, 2011). They have proved challenging to ground theoretically (see Beaudry and Portier, 2004 and 2007; Den Haan and Kaltenbrunner, 2009 and Jaimovich and Rebelo, 2009). They are also difficult to identify and there is empirical evidence both for and against these cycles (for: Beaudry and Portier, 2006; and Beaudry and Lucke, 2010; against: Barksy and Sims, 2011 and 2012). Lorenzoni (2007) studies the optimal monetary response to productivity news shocks and finds that policymakers should announce in advance how they will react to new information, which is consistent with my findings. Recent work has also studied news about future fiscal policy, and find that identifying anticipated shocks is crucial in understanding government tax and spending multipliers (Mertens and Ravn, 2010, 2011, 2012 and 2013; Romer and Romer, 2010; Ramey, 2011). Arezki et al. (2017) use oil discoveries as a well-identified news shock, and find that during the period between discovering and producing oil GDP and employment falls, and consumption rises financed by foreign borrowing, which are reversed when oil production begins. Using a parsimonious model I am able to replicate these dynamics, and extend their analysis by introducing nominal rigidities to study how monetary policy should respond.

Third, this paper adds to work that emphasizes the importance of the natural real interest rate in conducting monetary policy. The natural rate of interest (or "r star") was first proposed by Wicksell (1936), and is the real interest rate that would exist without nominal rigidities. The importance of this rate for conducting monetary policy is well-established, and has particularly influenced monetary policy in recent years (see Draghi, 2016; Yellen, 2017; Woodford, 2003; Laubach and Williams, 2003; Barksy et al., 2014; and Curdia et al., 2015; Holston, et al., 2017). The natural interest rate is typically expressed as a function of the expected change in natural output (Gali, 2008), so is intrinsically linked to news that natural output will change in the future, like oil discoveries. To my knowledge this is the first paper to emphasize the link between news shocks and the natural rate of interest when conducting monetary policy, both in general and for oil discoveries in particular.

The rest of the paper proceeds as follows. Section 2 develops a model of a small open economy with a government that receives oil revenues, and households that respond to news about the nation's oil wealth. Section 3 introduces the central bank's micro-founded loss function and derives optimal monetary policy. Section 4 uses stylized illustrations to demonstrate that optimal policy is important by comparing it to Taylor rules and an exchange rate peg. Section 5 then simulates how the model responds to the UK's North Sea oil discovery. Section 6 concludes.

2 Model

This section extends a standard, workhorse model of a developed small open economy (Gali and Monacelli, 2005 and 2008; Gali, 2008) with two key additions. The first is a government that receives a stream of foreign-denominated oil revenues, and spends them on home goods according to a simple fiscal rule. The second is households that respond to news about future oil production, which raises their wealth relative to abroad, and relaxes the "divine coincidence" that characterizes similar models.

2.1 Households

The representative household maximizes utility subject to a per-period budget constraint,

$$\begin{aligned} \max_{C_t,N_t,D_t} U_0 &= E_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\chi)\ln C_t + \chi \ln G_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right] && (2.1) \\ s.t. \qquad P_t C_t &\leq W_t N_t - P_{H,t} T_t - E_t[M_{t,t+1} D_{t+1}] + D_t && (2.2) \end{aligned}$$

where C_t is a domestic consumption bundle with consumer price index (CPI), P_t; G_t is the government spending bundle which is partly funded by taxes, T_t, levied in domestic currency, $P_{H,t}$; N_t is hours worked for wages, W_t; and D_{t+1} is the nominal payoff in domestic currency at time $t+1$ of the household's portfolio of perfectly internationally traded financial assets (including shares in firms), which has stochastic discount factor, $M_{t,t+1}$.[1] Utility will depend on three key parameters: the discount factor, β, the utility weight of government spending, χ, and the elasticity of labour supply, φ. I use lower case letters to denote the log of each variable, define the time discount rate as, $\rho \equiv \beta^{-1} - 1$, the nominal interest rate as, $i_t = ln(E_t[M_{t,t+1}]^{-1})$, and CPI inflation as, $\pi_t \equiv p_t - p_{t-1}$. This optimization problem therefore yields two log-linearized

[1] This abstracts from borrowing constraints that may be faced by developing countries (see van der Ploeg and Venables, 2011; Wills, 2017).

first order conditions, describing labour supply and the stochastic Euler equation,[2]

$$w_t - p_t = c_t + \varphi n_t - \ln(1-\chi) \tag{2.3}$$

$$c_t = E_t[c_{t+1}] - (i_t - E_t[\pi_{t+1}] - \rho) \tag{2.4}$$

Total consumption is a Cobb-Douglas bundle of home (H) and foreign (F) goods, $C_t \equiv C_{H,t}^{1-\alpha} C_{F,t}^{\alpha} \cdot (1-\alpha)^{-(1-\alpha)}\alpha^{-\alpha}$. The preference for foreign goods is described by an index of openness, $\alpha \in [0,1]$. Home and foreign goods are allocated optimally to minimize, $P_t C_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$, where CPI is an index of home and foreign prices, $P_t = P_{H,t}^{1-\alpha}P_{F,t}^{\alpha}$. Note that the elasticity of substitution between home and foreign goods equals one. If home and foreign consumers were also perfectly symmetric, then there would be a "divine coincidence" in the objectives of a central bank in a closed and an open economy. This happens because any movement in the terms of trade will see the increase in purchasing power over imports perfectly offset by lower demand for exports. Divine coincidence breaks down in our model because of asymmetric wealth, as discussed in Section 2.5.

Home and foreign goods are both CES bundles of individual varieties that are produced in all countries. The foreign good is a bundle of goods produced by a continuum of foreign countries, $f \in [0,1]$, with an elasticity of substitution of one. In logs this is, $c_{F,t} = \int_0^1 c_{f,t} df$, with price index, $p_{F,t} = \int_0^1 p_{f,t} df$. The consumption good produced at home and in every foreign country is a CES bundle of individual varieties, $i \in [0,1]$, $C_{j,t} \equiv \left(\int_0^1 C_{j,t}(i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$ for $j = [H, f \in [0,1]]$. Each variety is produced in every country and has an elasticity of substitution of ϵ, allowing for monopolistic price setting in Section 2.6. The associated price indices are, $P_{j,t} \equiv \left(\int_0^1 P_{j,t}(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$ for $j = [H, f \in [0,1]]$.

2.2 Terms of Trade and Exchange Rates

The effective non-oil terms of trade (the "terms of trade" from now on) allocates global household demand between home and foreign goods, and is defined as the price of the foreign bundle in terms of home goods, $s_t \equiv p_{F,t} - p_{H,t}$ (see Gali and Monacelli, 2005). The terms of trade links the CPI to the domestic price level, $p_t = p_{H,t} + \alpha s_t$, so CPI inflation is $\pi_t = \pi_{H,t} + \alpha \Delta s_t$. The price index for all goods produced in any foreign country expressed in the home currency is,

[2]The model is log-linearized around a steady state for tractability. This is appropriate because in the developed countries that are the focus of this paper, like Australia, Canada, the UK and the US, resource rents fluctuate by less than 5 percent of GDP (World Bank, 2015). In Norway they account for 20 percent of GDP, but approximately 6 percent is released from the sovereign wealth fund each year.

$p_{F,t} = e_t + p_t^*$, where $e_t \equiv \int_0^1 e_{f,t} df$ is the effective nominal exchange rate and, $p_t^* \equiv \int_0^1 p_{f,t} df$, is the world price index. The terms of trade is thus related to the nominal exchange rate by, $s_t = e_t + p_t^* - p_{H,t}$. The bilateral real exchange rate describes the CPI in country f as a proportion of the CPI at home, $q_{f,t} \equiv \ln \mathcal{Q}_{f,t} = e_t^f + p_t^f - p_t$, where p_t^f is the (log) CPI in country f. The effective real exchange rate is an index of the bilateral real exchange rates, $q_t \equiv \int_0^1 q_{f,t} df$. Linking the effective real exchange rate and the terms of trade gives, $q_t = \int_0^1 (e_t^f + p_t^f - p_t) df = e_t + p_t^* - p_t = (1-\alpha)s_t$.

2.3 Oil Production

When oil is discovered there is a delay before extraction begins and the associated rents are spent.[3] I want to capture this in the simplest and most tractable way possible, so I construct a profile for oil production that jumps V periods after discovery, and then is constant until the new reserves are exhausted. This can be thought of as oil production following a Leontief function of the stock of capital, K_t, and known oil reserves, R_t, $O_t = \min(aK_t^c, bR_t)$.[4] Known reserves increase randomly with discoveries, $\epsilon_{R,t}$, and fall when fields are exhausted E_t, $R_t = R_{t-1} - E_t + \epsilon_{R,t}$. When oil is discovered I assume it takes V periods for the government-owned extraction firm to build the capital needed to extract it, financed offshore.[5] Until then oil output remains constant, so $K_t = K^s$ if $t < V$ and $K_t = \frac{b}{a} R_t^{1/c}$ if $t \geq V$. During this phase there is a flow of investment demand for home and foreign goods, $J_t = J_{H,t}^{1-\alpha} J_{F,t}^{\alpha} \cdot (1-\alpha)^{-(1-\alpha)} \alpha^{-\alpha}$ at price P_t for $t \in [0, V]$ and zero otherwise.[6] There are decreasing marginal returns to investment each period, $K_t = (1-\delta)K_{t-1} + \ln J_t$, to tractably encourage investment smoothing, and it is financed using the internationally traded financial asset, $P_t J_t = -E_t[M_{t+1} B_{t+1}] + B_t$. The extraction firm must therefore maximize the intertemporal effectiveness of its investment over

[3]For oil and gas discoveries, of the 400 offshore fields discovered in the UK between 1957-2011, the mean time between discovery and production was 4.5 years (OGA, 2017), similar to the 4-6 years found by Arezki et al. (2017). For recent US shale gas the delay was 2-9 years. For mineral projects, of the 82 new projects in Western Australia during the mining boom of October 2012 the mean expected time to production was approximately 3 years (BREE, 2012). Of the 35 new projects that were "committed but not yet complete", only 6 percent expected to begin production within 3 months, and only 60 percent expected to begin production within 15 months.

[4]Over the life of the well there may be some substitution where better capital allows the exploitation of more marginal reserves. However, when oil is discovered a partially-built well will produce nothing, which is what the Leontief function captures here.

[5]In practice the time to build the oil wells will endogenously vary with the complexity of the site, the costs of capital and labour, and the oil price (see for example Pindyck, 1978; Gross and Hanson, 2018). I assume this time is exogenous to keep the model tractable - which allows a closed-form solution, and to provide a clean stylised depiction of a news shock - which is the focus of the paper.

[6]This uses the same degree of openness as the consumption bundle, which provides major advantages in keeping the model tractable enough to arrive at a closed-form solution.

V periods, taking into account movements in the terms of trade,

$$\begin{aligned}
\max_{J_t, B_t} \quad & E_t[\sum_{v=0}^{V} \beta^{t+v} \left(\ln J_{t+v} + \lambda_{t+v}(-M_{t+v,t+v+1}B_{t+v+1} + B_{t+v} - P_{t+v}J_{t+v})\right)] \\
\text{so } M_{t,t+1} \quad = \quad & \beta \left(J_t/J_{t+1}\right) \left(P_t/P_{t+1}\right) \qquad (2.5)
\end{aligned}$$

Once oil production begins we assume it is constant until exhaustion for tractability, $O_t = bR_t$ for $t \geq V$ and $O_t = O_0$ otherwise, $bR_t > O_0 > 0$.[7] The production firm receives an inelastic, foreign denominated stream of revenues[8] each period, $\varepsilon_t P_{O,t} O_t$, where the oil price $P_{O,t}$ is expressed in units of the global price index and numeraire $P_t^* = 1$, so that fluctuations in the exchange rate, ε_t, will alter the relative value of oil income. The firm's costs are covered using $(1 - \tau_O)$ of oil revenues, and the rest is transferred to the government.

2.4 Government

The government receives lump-sum oil rents from the government-owned extraction firm, levies lump-sum taxes and spends the proceeds on public consumption and a small transfer to firms.[9] I abstract from any distortionary taxes. This paper's first addition to the benchmark model is that the government will spend oil revenues according to a simple rule. This is summarized in the government's per-period budget constraint, where F_t is the portfolio of risk-free foreign assets held at time t, T_t are lump sum taxes, τ_t is a subsidy to firms and $\tau_O \varepsilon_t P_{O,t} O_t$ are oil revenues,

$$P_{H,t}G_t + \tau_t + E_t[M_{t,t+1}F_{t+1}] \leq F_t + \tau_O \varepsilon_t P_{O,t} O_t + P_{H,t}T_t. \qquad (2.6)$$

The government controls how oil revenues are released into the economy, which is summarized by the "resource balance", RB_t (Pieschacon, 2012). The resource balance is expressed in units of world currency as, $RB_t = \tau_O P_{O,t} O_t + F_t - E_t[M_{t,t+1}F_{t+1}]$. Substituting this into 2.6 gives, $P_{H,t}G_t + \tau_t = \varepsilon_t RB_t + T_t$, which in log-linear terms is, $\hat{g}_t = \hat{s}_t - \hat{p}_t^* + \widehat{rb}_t - \hat{t}_t$, where $t_t = -\ln(1 - \frac{T_t}{G_t})$. Foreign prices, and lump sum taxes as a share of government spending, are both assumed to be constant, so $\hat{g}_t = \hat{s}_t + \widehat{rb}_t$. Assuming that the share, rather than the level, of lump sum taxes is constant makes the analysis more tractable, which is discussed in Appendix D.

[7] Having non-zero oil production before and after the boom, $O_0 > 0$, is necessary to avoid log-linearizing around a zero value in the initial steady state.

[8] Oil is inelastically supplied over business-cycle horizons because of geological constraints (Anderson at al., 2014), and are regularly treated as an exogenous windfall of foreign exchange (van der Ploeg and Venables, 2011).

[9] National oil companies control 90% of the world's oil reserves and 75% of production (Tordo et al., 2011). In theory governments could sell future oil revenues when they are discovered. In practice this does not happen, due to the size of the assets and the political risk if oil prices rise.

Government revenues are spent on public consumption and a small transfer to firms. The government only consumes home-produced goods, reflecting the higher weight of services in public consumption like health care, education and justice. These goods are combined in a CES bundle with the same elasticity of substitution as for households, $G_t \equiv \left(\int_0^1 G_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$. Quantities are chosen optimally so government demand for each good is, $G_t(i) = (P_{H,t}(i)/P_{H,t})^{-\epsilon} G_t$. The government also makes a small transfer to firms, τ_t, to offset the marginal cost distortion from monopolistic competition, following Gali and Monacelli (2005).

The government will follow one of two simple fiscal rules, illustrated in Section 4. The first spends all oil revenues as they are received. This neatly captures the news shock we are interested in. It is also relevant in practice as many countries still feed resource revenues directly into government budgets (including Australia, Canada, Ecuador, Indonesia, Norway pre-1990 and the UK). The second is a Bird in Hand rule where expenditure is a fixed proportion of sovereign wealth fund assets, as followed by Norway post-1990. In contrast, the optimal fiscal response to an oil discovery would be to immediately increase spending by the oil's permanent annuitised value. I do not study this case because it removes the "news" aspect of the discovery - the focus of this paper - and becomes a simple unanticipated demand shock (see Gali, 2008 for example).

2.5 International Risk Sharing

The international risk sharing condition lets us express domestic consumption and investment as a function of world consumption. I assume that claims on all non-oil shocks are perfectly traded internationally. This isolates oil discoveries, which increase the wealth of home households relative to foreigners. Using this assumption, equations 2.4 and 2.5, and identical preferences across countries we have the following for every country, f,

$$E_t\left[\beta\left(\frac{C_t}{C_{t+1}}\right)\left(\frac{P_t}{P_{t+1}}\right)\right] = E_t\left[M_{t,t+1}\right] = E_t\left[\beta\left(\frac{C_t^f}{C_{t+1}^f}\right)\left(\frac{P_t^f}{P_{t+1}^f}\right)\left(\frac{\varepsilon_t^f}{\varepsilon_{t+1}^f}\right)\right] \tag{2.7}$$

$$= E_t\left[\beta\left(\frac{J_t}{J_{t+1}}\right)\left(\frac{P_t}{P_{t+1}}\right)\right] \tag{2.8}$$

Households in each country also face a transversality condition. Everything they earn must eventually be consumed, so $\lim_{T\to\infty} M_{0,T} D_T = 0$. Summing 2.2 over an infinite horizon for both home and foreign countries, and using 2.4 (see Appendix A.1) gives the following expressions tying domestic consumption, C_t, to foreign consumption, C_t^f, adjusted for relative household wealth, Θ_t^f, and the real exchange rate, $\mathcal{Q}_{f,t}$,

$$C_t = \Theta_t^f C_t^f \mathcal{Q}_{f,t} \tag{2.9}$$

$$\text{where } \Theta_t^f = \frac{E_t[\sum_{s=0}^{\infty} M_{t,t+s}\left(W_{t+s}N_{t+s} - P_{H,t+s}T_{t+s}\right)] + D_t}{E_t[\sum_{s=0}^{\infty} M_{t,t+s}^f\left(W_{t+s}^f N_{t+s}^f - P_{H,t+s}^f T_{t+s}^f\right)] + D_t^f} \tag{2.10}$$

Taking logs, integrating over all countries f, and using $c_t^* = \int_0^1 c_t^f df$, gives the following, where $\vartheta_t \equiv \ln \Theta_t$,

$$c_t = \vartheta_t + c_t^* + (1-\alpha)s_t \tag{2.11}$$

This paper's second addition to the benchmark model is that relative household wealth, ϑ_t, increases immediately after an oil discovery. This describes the expected net present value of domestic household income, relative to households abroad, and is always constant in expectation, $E_t[\Theta_{t+s}] = \Theta_t \forall s \geq 0$. However, when domestic households hear news of an oil discovery they will anticipate higher income in the future. Their consumption will immediately jump accordingly, so news about the future has effects today. The change in (log) relative household wealth after an oil discovery will be proportional to the present discounted value of the government's resource-balance, as shown in Appendix A.1,

$$\hat{\vartheta}_0 = \tfrac{(1-\beta)(\gamma_G-\chi)}{(1-\chi)-(1-\gamma_G)(1-\alpha)} E_0\left[\sum_{t=0}^{\infty} M_{0,t} \widehat{rb}_t\right]. \tag{2.12}$$

Uncovered interest parity will also hold, $E_t[q_{t+1}] - q_t = r_t - r^*$. This follows from 2.4 at home and abroad, $E[\Delta c_{t+1}] = E[\Delta c_{t+1}^*] + (1-\alpha)E[\Delta s_{t+1}]$ (from equation 2.7), and the definition of the real interest rate, $r_t \equiv i_t - E[\pi_{t+1}]$.

Similarly, combining 2.8 and the total capital needed to extract a new oil discovery, $B_{t+V} = \frac{b}{a}R_t^{1/c}$, links the flow of investment to world consumption and the terms of trade,

$$J_t = \Theta_{J,t}^f C_t^f \mathcal{Q}_{f,t} \tag{2.13}$$

$$\text{where } \Theta_{J,t}^f = \frac{M_{t,t+V}(\frac{b}{a}R_t^{1/c})}{E_t[\sum_{s=0}^{\infty} M_{t,t+s}^f\left(W_{t+s}^f N_{t+s}^f - P_{H,t+s}^f T_{t+s}^f\right)] + D_t^f} \tag{2.14}$$

$$\text{integrating (in logs) } j_t = \vartheta_{J,t} + c_t^* + (1-\alpha)s_t. \tag{2.15}$$

When oil is discovered, $\epsilon_{R,t} > 0$, the total amount of extraction capital needed will jump, $\Theta_{J,t} \uparrow$.

Investment in all periods $t \in [0, V]$ will jump, and will be allocated efficiently between periods based on movements in the real exchange rate, Q_t.

2.6 Firms

Each domestic firm will produce a differentiated good using labour and a common technology, $Y_t(i) = A_t N_t(i)$. Oil is not included as a factor of production, to focus on the windfall effects of a resource discovery. These firms do not use capital for tractability, following standard models (Gali and Monacelli, 2005).

Firms set prices in a staggered way according to Calvo (1983). A measure of $(1-\theta)$ randomly selected firms set new prices in each period, with an individual firm's probability of re-optimizing in any given period being independent of the time elapsed since it last reset its price. In a standard result, derived in Appendix A.3, the optimal price-setting strategy for the typical firm resetting its price in period t can be approximated by the log-linear rule,

$$p^*_{H,t} = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t[mc_{t+k} + p_{H,t}] \tag{2.16}$$

where $p^*_{H,t}$ denotes the log of newly set domestic prices, and $\mu \equiv \ln\left(\frac{\epsilon}{\epsilon-1}\right)$ is the log of the optimal markup in the flexible price economy. The pricing decision is thus forward-looking, as firms recognize that the price they set will last for a random number of periods. In the flexible price limit (as $\theta \to 0$) we recover the markup rule $p^*_{H,t} = \mu + mc^n_t + p_{H,t}$, so that the marginal cost in the flexible price state is $mc^n_t = -\mu$.

2.7 Equilibrium Dynamics

Equilibrium is characterized by standard aggregate demand (IS curve) and aggregate supply (Phillips curve) conditions, which describe dynamics around a steady state. Output begins at this (log) steady state level, $y^s \equiv \ln Y^s$. After oil is discovered the actual level of output will deviate from the steady state, $\hat{y}_t \equiv y_t - y^s$. So too will the "natural" (flexible-price) level of output, $\hat{y}^n_t \equiv y^n_t - y^s$ (unlike Gali and Monacelli, 2005). The output gap is the difference between the two, $\tilde{y}_t \equiv y_t - y^n_t$.

2.7.1 Aggregate Demand

The market clearing condition for each domestically produced good requires output to equal demand from domestic consumption and investment, government purchases, and consumption from each foreign country. In Appendix A.2 this market clearing condition is combined with the Euler equation (2.4) to give the standard IS curve,[10]

$$\begin{aligned} \tilde{y}_t &= E_t[\tilde{y}_{t+1}] - (i_t - E_t[\pi_{H,t+1}] - r_t^n) && (2.17) \\ r_t^n &= \rho + (1+\varphi)E_t[\Delta a_{t+1}] - \varphi E_t[\Delta \hat{y}_{t+1}^n] \\ &= \rho + E_t[\Delta \hat{s}_{t+1}^n] + E_t[\Delta \hat{\vartheta}_{t+1}^n] + E_t[\Delta \hat{c}_{t+1}^*] && (2.18) \end{aligned}$$

The natural interest rate in equation 2.18 plays a central role in this paper. It is the Wicksellian interest rate at which monetary policy is neither stimulatory nor contractionary, which would hold in the absence of nominal rigidities. It rises with expected improvements in technology (as in Gali and Monacelli, 2005), but falls with expected increases in natural output for reasons other than technology, like oil. This happens because of the terms of trade.[11] An anticipated non-technological increase in $\hat{y}_t^n$ appreciates the terms of trade, which tightens household budget conditions, and lowers the returns to saving, $\hat{r}_t^n$. The natural level of output, $\hat{y}_t^n$, is derived in Appendix A.3 from marginal costs when prices are flexible,

$$\hat{y}_t^n = \hat{a}_t + \tfrac{\gamma_G}{1+\varphi}\left(\widehat{rb}_t - \hat{y}_t^*\right) + \tfrac{(1-\alpha)(1-\gamma_G)}{1+\varphi}\hat{\vartheta}_{J,t} - \tfrac{\alpha+(1-\alpha)\gamma_G}{1+\varphi}\hat{\vartheta}_t \qquad (2.19)$$

Natural output increases in technology, $\hat{a}_t$, and government spending (the resource balance), $\widehat{rb}_t$, but falls in foreign output, $\hat{y}_t^*$. The influence of the latter two is determined by the extent the government, γ_G, introduces asymmetry between home and abroad (see Gali and Monacelli, 2008). Natural output also increases with oil investment, $\hat{\vartheta}_{J,t}$, but falls with relative household wealth, $\hat{\vartheta}_t$, as richer households consume leisure and higher domestic prices lead to substitution away from home goods.

[10]The IS curve takes this form due to the symmetry between consumption and investment, and the way government spending is characterized, $\hat{g}_t = \hat{s}_t + \widehat{rb}_t$.

[11]In expectation $E_t[\Delta \hat{\vartheta}_{t+1}^n] = 0$ as it represents the present discounted value of all known future household income. In this paper we also hold $\hat{c}_{t+1}^*$ constant.

2.7.2 Aggregate Supply

Firms set prices after observing the demand schedule of households and the government. In a standard result, derived in Appendix A.3, aggregate supply can be approximated to the first order by the standard New Keynesian Phillips curve,

$$\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \lambda(1+\varphi)\tilde{y}_t. \tag{2.20}$$

Appendix A.3 also gives a second-order approximation of aggregate supply, which is used to derive the central bank's loss function (see Benigno and Woodford, 2005)

2.7.3 The Steady State

The steady state in the home country, when the home government receives oil revenues while foreign governments do not, and all countries are otherwise symmetric with $A^i = 1$, is as follows, as proved in Appendix A.4,

$$\begin{array}{lllll} N^s = \left(\frac{1-\chi}{1-\gamma_T}\right)^{1/(1+\varphi)} & ; & \Theta^s = \frac{(1-\gamma_T)(\alpha+(1-\alpha)\Theta^s_J)}{(1-\gamma_G)-(1-\alpha)(1-\gamma_T)} & ; & \Theta^s_J = \frac{\gamma_J}{1-\chi}A^s N^s (S^s)^{\alpha-1} \\ S^s = (N^s)^{-\varphi}(\Theta^s)^{-1} & ; & C^s = \Theta^s(1-\chi)(S^s)^{1-\alpha} & ; & Y^s = A^s N^s \end{array} \tag{2.21}$$

where $\gamma_J \equiv J^s_H/Y^s$, $\gamma_G \equiv G^s/Y^s$, $\gamma_T \equiv T^s/Y^s$ and $(\gamma_G - \gamma_T)$ is the steady state share of government spending financed by oil revenues, expressed as a proportion of GDP.

The steady state accounts for permanent differences in wealth due to oil income. For intuition, the household wealth ratio is greater than one if there is oil investment, or if government spending is partly financed by oil revenues at home but not abroad: $\Theta^s > 1$ if $\Theta^s_J = 0 \cup \gamma_G > \gamma_T$ or if $\Theta^s_J > 0 \cup \gamma_G = \gamma_T$. This will also lead to an appreciated steady state terms of trade, $S < 1$, and relatively higher home consumption, $C^s > C^*$.

3 Monetary Policy

3.1 Central Bank Objective

The central bank's objective is derived from household welfare and involves stabilizing domestic inflation and the output gap with an additional term that accommodates changes in the natural level of output, as stated in the following proposition.

Proposition 1. *If the government spends oil revenues according to the fiscal rule,* $\hat{g}_t = \hat{s}_t + \widehat{rb}_t$*, then a second order approximation of the central bank's loss function is,*

$$\mathbb{L}_0 \quad = \quad E_0 \sum_{t}^{\infty} \beta^t \left\{ \tfrac{1}{2}\phi_\pi \pi_{H,t}^2 + \tfrac{1}{2}\phi_y \tilde{y}_t^2 + (1+\varphi)\hat{y}_t^n \tilde{y}_t \right\} + o(\tilde{y}_t^3) + t.i.p. \tag{3.1}$$

where $\phi_\pi = \frac{\epsilon}{\lambda} + \frac{\alpha(1-\chi)(1+\theta-\epsilon\theta\varphi)}{\lambda\theta(1+\varphi)}$, $\phi_y = (1+\varphi)(1+\alpha(1-\chi))$ *and t.i.p. are terms independent of policy.*

Proof. See Appendix B.1. □

The central bank has an incentive to stabilize domestic inflation, $\pi_{H,t}^2$, and in turn the output gap, $\tilde{y}_t^2$. This result will be familiar from standard closed economy models with Calvo pricing and an constant employment subsidy that offsets the distortion from monopolistic competition (e.g. Rotemberg and Woodford, 1999). In such a setting the only distortion is from sticky prices, so there is a "divine coincidence" where stabilizing prices is equivalent to stabilizing the welfare-relevant output gap (Monacelli, 2013).

In addition the central bank also has an incentive to influence the level of the output gap, $\tilde{y}_t$. This is because the central bank in an open economy can manipulate the terms of trade (and therefore output, $\tilde{s}_t = \tilde{y}_t$) to the advantage of domestic consumers (Corsetti and Pesenti, 2001).[12] Some authors choose to neutralize this effect with a second ad hoc employment subsidy that exactly offsets the terms of trade distortion, restoring divine coincidence and ensuring the natural level of output stays constant at the optimal level (see Gali and Monacelli, 2005). I take a different approach as I am explicitly interested in how the natural level of output changes when oil is discovered, and use a second-order approximation of aggregate supply when deriving the loss function (Benigno and Woodford, 2005). During an oil boom the natural level of output will exceed its steady state ($\hat{y}_t^n > 0$), so the central bank will have an incentive to appreciate the terms of trade to reduce output and increase leisure ($\tilde{y}_t \downarrow$).

3.2 Optimal Policy

When oil is discovered the central bank should respond in advance to anticipated changes in the natural level of output. Assuming that the central bank can credibly commit to future policy, and operates from a timeless perspective,[13] the analytical solution for the path of prices, output

[12] If home and foreign goods are not perfect substitutes and monetary policy is non-neutral.

[13] This is necessary when using a second-order approximation of the loss function (Benigno and Woodford, 2005)

and optimal policy is as follows.

Proposition 2. *For a monetary authority conducting policy from a timeless perspective, the optimal paths for $\hat{p}_{H,t}$, $\hat{y}_t$ and i_t after an oil discovery are,*

$$\hat{p}_{H,t} = \delta \hat{p}_{H,t-1} - b\delta E_t \left[\sum_{s=0}^{\infty} (\beta\delta)^s \hat{y}^n_{t+s} \right] \tag{3.2}$$

$$\tilde{y}_t = -\tfrac{\lambda\phi_\pi}{(1+\alpha(1-\chi))} \hat{p}_{H,t} - \left(\tfrac{1}{(1+\alpha(1-\chi))} + 1 \right) \hat{y}^n_t \tag{3.3}$$

$$(i_t - \rho) = c_1 \hat{p}_{H,t} + c_2 E_t[\Delta a_{t+1}] - c_3 E_t[\Delta \hat{y}^n_{t+1}] + c_4 E_t \left[\sum_{s=0}^{\infty} (\beta\delta)^s \hat{y}^n_{t+1+s} \right] \tag{3.4}$$

where $\delta = (1 - \sqrt{1 - 4a^2\beta})/(2a\beta)$, $c_1 = \left(\tfrac{\lambda\phi_\pi}{(1+\alpha(1-\chi))} - 1 \right)(1-\delta)$, , $c_2 = (1+\varphi)$ $c_3 = \tfrac{1}{(1+\alpha(1-\chi))} + \varphi$ *and* $c_4 = \left(\tfrac{\lambda\phi_\pi}{(1+\alpha(1-\chi))} - 1 \right) \left(\tfrac{\lambda(1+\varphi)}{(1+\alpha(1-\chi))} \right) \delta$.

Proof. See Appendix B.2. □

When oil is discovered, a credible central bank should optimally respond to the domestic price level and expected changes in the natural level of output. It responds to the domestic price level, $c_1 \hat{p}_{H,t}$, as in standard small open economy models (e.g. Gali and Monacelli, 2005). The rule gives a unique and determinate solution when $c_1 > 0$ or $\lambda\phi_\pi > (1 + \alpha(1-\chi))$. Monetary policy will have a persistent response to shocks by committing to target the level rather than the change in prices, as would happen under discretion.

Optimal monetary policy should also respond to expected changes in natural output, $-c_2 E_t[\Delta \hat{y}^n_{t+1}]$. This accommodates the fall in the natural rate of interest as output is expected to increase, so the returns to saving are lower. It also ensures that the required increase in domestic prices relative to abroad happens entirely through the nominal exchange rate in the future. Optimal policy also offsets the recursive effects of future demand on forward-looking inflation, $+c_3 E_t\left[\sum_{s=0}^{\infty} (\beta\delta)^s \hat{y}^n_{t+1+s}\right]$. Under standard calibrations this effect is less important, $c_2 \gg c_3$. These effects are illustrated in the next section.

This intuition extends to more gradual increases in oil-financed expenditure, as illustrated in Section 4.2. Indeed the intuition also extends more generally to anticipated changes in government spending that would change the natural level of output, such as pre-announced budget plans (see Ramey, 2011). Finally, it is straightforward to see that the results in proposition 2 collapse to the benchmark case of Gali and Monacelli (2005) if an employment subsidy is used to hold the natural level of output constant.

3.3 Other Monetary Rules

The importance of optimal policy can bee seen by comparing it to three ad-hoc monetary rules: an optimal Taylor rule, a naive Taylor rule and an exchange rate peg (the latter used by three-quarters of oil exporters in practice).

The optimal Taylor rule sets interest rates in response to domestic inflation, the output gap and the natural rate of interest as given in equation 3.5. As the natural rate of interest is a function of the expected change in the natural level of output (equation 2.18) it plays a similar role to the $E_t[\Delta \hat{y}^n_{t+1}]$ term in equation 3.4. The naive Taylor rule has interest rates respond to domestic inflation and the output gap only, as given in equation 3.6. This ignores the effects of oil and naively assumes that the natural rate of interest remains constant, as is done widely in practice (Taylor, 1993 and Woodford, 2001).

$$i_t = \theta_\pi \pi_{H,t} + \theta_y \tilde{y}_t + r^n_t \tag{3.5}$$

$$i_t = \theta_\pi \pi_{H,t} + \theta_y \tilde{y}_t + \rho \tag{3.6}$$

The exchange rate peg is maintained using the nominal interest rate. In this model the capital account is open and international markets in financial assets are partially complete (except for claims to national oil wealth, see section 2.5). Any accumulation of foreign currency reserves by the central bank can therefore be offset by households. As a result the nominal exchange rate must be stabilized by matching the nominal interest rate at home to that abroad. The dynamics of the economy under and exchange rate peg are derived explicitly in Appendix B.3.

4 Stylized Illustrations

4.1 Spend-All Rule

To illustrate how discoveries affect monetary policy I calibrate the model to represent a typical small open economy with natural resources (following Gali and Monacelli, 2005; see Appendix C). In the steady state the resource balance is 5% of GDP, which is broadly consistent with Australia, Canada, Norway, the UK, and the US (see footnote 5). Oil with a net present value of 50% of GDP is discovered at $t = 0$, which is followed by 16 quarters when investment approximately doubles from its starting point of 3% of GDP, and then extraction which permanently increases

Welfare Loss Relative to Optimal

	Optimal	Flexible	TR*	TR	Peg
Welfare Loss ($\hat{c}_0$ units)	-0.007%	0%	0%	1.04%	12.51%

Table 4.1: The welfare loss from an anticipated change in oil production under a spend-all rule (see figures 4.1 and 4.2), expressed as the amount of additional consumption needed at $t = 0$, $\hat{c}_0$, to replicate welfare under flexible prices.

oil's share of GDP by 2.5%.[14] I assume that the government spends all oil revenues as they are received for clarity; a Norwegian-style Bird-in-Hand rule with a temporary windfall is studied in Section 4.2.[15] Taylor rules are compared using a typical parameterization of $\phi_\pi = 1.5$ and $\phi_y = 0.5$. The punchline is that when the government starts spending oil revenues it will temporarily lower the natural interest rate, because it is anticipated. Optimal monetary policy should track the natural interest rate.

4.1.1 Baseline: Flexible Prices

As a baseline I first consider the case of flexible prices to show that oil discoveries affect the economy in two stages: at discovery and at production ("*Flex*" in Figure 4.1).

First, when oil is discovered ($t = 0$) households learn they are wealthier[16] and so consume more, and the extraction firm invests in wells. Wealthier households also demand more leisure, so the initial net increase in natural output is only small. Instead, the additional demand from consumption and investment is met by inflation (a terms of trade/real exchange rate appreciation), and more imports financed by foreign borrowing. The terms of trade appreciation reduces government consumption of home goods. The real natural interest rate (labelled r^n in the "Nominal Interest Rate" panel) temporarily drops in the quarter before oil production begins (equation 2.18) as agents anticipate an increase in natural output and in turn lower returns to saving.

Second, when production begins ($t = 16$) the government starts receiving oil revenues and spends a share on domestic goods. The additional government demand for home goods causes inflation to rise again, further appreciating the terms of trade. This is partially offset by the

[14] Calibration is consistent with the mean discovery size and production delay for onshore oil wells in Arezki et al. (2017).

[15] I do not consider the case of the government borrowing against future oil revenues to immediately raise spending (according to the permanent income hypothesis) as this is rarely seen in practice and would remove the news-shock characteristics which are the focus of this paper.

[16] Households anticipate that the goverment will spend oil revenues on domestic goods in the future, in turn raising the wages earned by households.

end of drilling investment, which now falls below its initial level due to the real appreciation. The real appreciation causes domestic households to consume less overall, repaying their foreign borrowing from the first stage, and substitute from home to foreign goods (as do foreign households). The net effect is an increase in domestic output because the government only consumes home goods, like education, health care and justice, while households can substitute to imports.

These real dynamics match the empirical evidence on oil discoveries. When oil is discovered, Arezki et al. (2017) find that private consumption and investment both jump immediately. Investment declines before production begins, but consumption remains elevated. During this period the current account declines for five years, and then starts rising when oil production begins. The real exchange rate (the terms of trade in this model) appreciates during the first five years after discovery. Toews et al. (2016) also find this, and show that it happens through rising prices of non-traded goods. Arezki et al. (2017) also find that government spending does not respond to the initial discovery, but jumps when production begins. Aggregate GDP similarly does not increase until production begins, peaking at approximately 3%, 8 years after discovery.

4.1.2 Exchange Rate Peg

I now consider a nominal exchange rate peg, used by 75% of resource-dependent economies,[17] to show how it can cause stagflation when oil is discovered ("*Peg*" in Figure 4.1).

When oil is discovered the terms of trade must appreciate, as outlined above. If the currency is pegged, then this must happen entirely through domestic inflation, which remains high throughout the anticipation period.[18] As prices are slow to adjust, output, consumption and foreign borrowing will initially overshoot their natural levels, inducing large fluctuations. Nominal rigidities will also cause prices to disperse between varieties of goods, leading to real distortions. The nominal interest rate remains unchanged, in order to maintain the peg.

When oil production begins, the terms of trade must appreciate for the second time due to the influx of government demand. This can also only happen through domestic inflation, and is anticipated by firms. As firms can only change prices randomly according to Calvo, they will start raising them in advance if given the opportunity. Thus, firms will have two incentives to raise prices between discovering and producing oil: retrospectively to deal with extra consumption and investment, and prospectively due to anticipated government demand.

[17]Excluding the proposed East African Monetary Union, see Baunsgaard et al. (2012)

[18]This is consistent with the empirical evidence in Toews et al. (2016), who attribute the entire real appreciation after an oil discovery to inflation of non-traded goods. While they do not control for monetary regime, it reflects the fact that most resource-dependent economies peg their currencies.

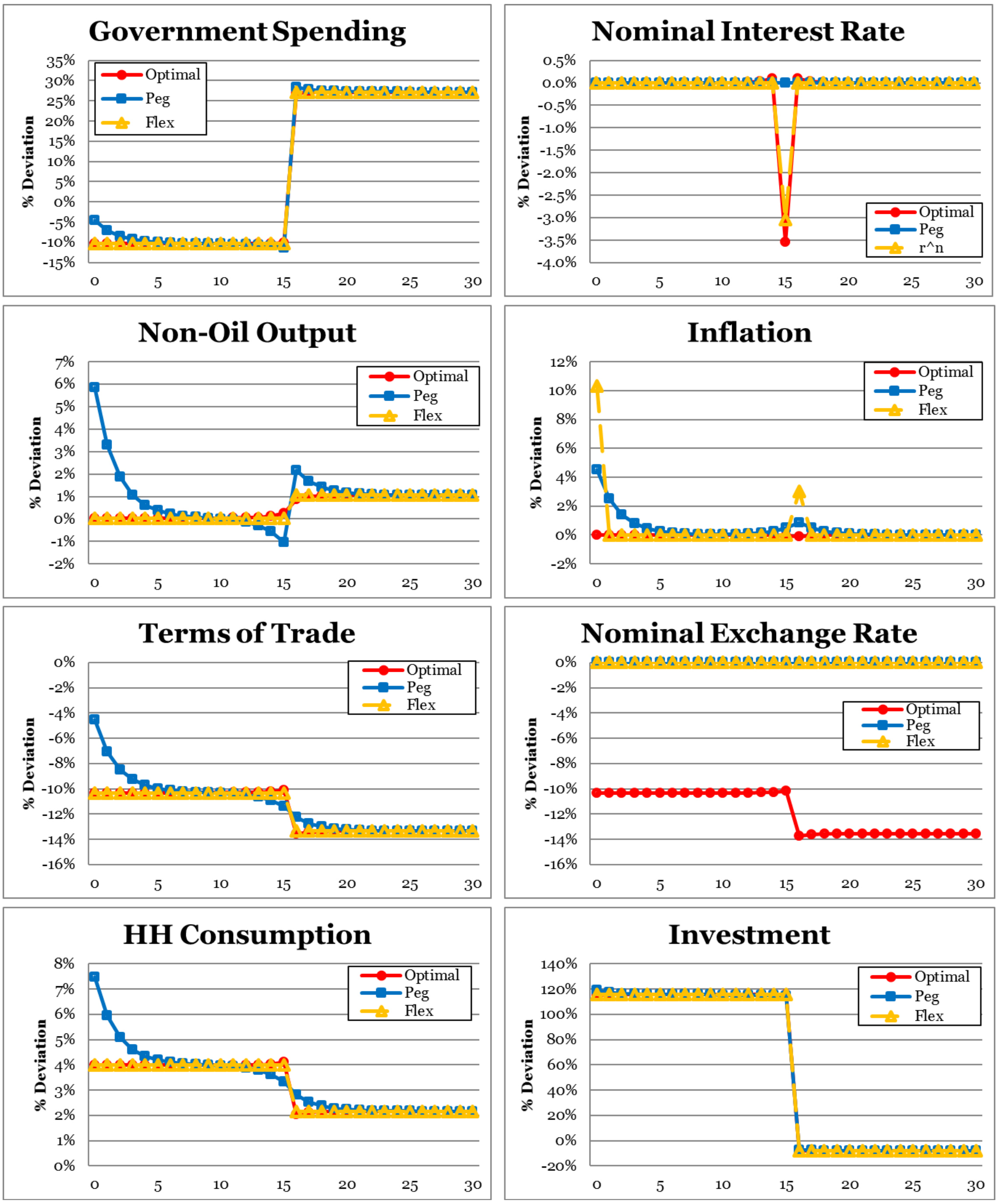

Figure 4.1: Forecast mean responses of key variables to an anticipated oil windfall, if the government spends oil revenues as they are received. Three monetary scenarios are illustrated: flexible prices (Flex), a pegged currency (Peg) and optimal policy from a timeless perspective (Optimal). Horizontal axis shows time in quarters.

The result is stagflation, with high inflation for an extended period causing output to fall below its natural level (a negative output gap, $\hat{y}_{15}^{peg} < \hat{y}_{15}^{flex}$ in Figure 4.1) before oil production begins.

Output only recovers when government spending begins. It will again jump too far, as all firms will not have had an opportunity to raise their prices in advance. An oil discovery under a currency peg will therefore lead to an extended period of inflation and large fluctuations in the output gap. If the oil discovery raises natural output by 1%, the output gap ($\tilde{y}_t^{peg} \equiv \hat{y}_t^{peg} - \hat{y}_t^{flex}$) may be a similar size. This reduces welfare by more than any other policy considered, as shown in Table 4.1. Therefore fixed exchange rates remove an important "shock absorber" for oil-exporters, as they force large changes in the terms of trade to happen through domestic inflation.

4.1.3 Optimal Policy

Optimal monetary policy prevents the large and persistent fluctuations in output, inflation and the terms of trade that follow an oil discovery under an exchange rate peg. To do this it allows the nominal exchange rate to appreciate at discovery, and tracks the temporary fall in the natural interest rate when production begins.

When oil is discovered the terms of trade must appreciate. Optimal policy allows this to happen via the nominal exchange rate rather than domestic prices, to avoid the sticky price distortions in the latter. This does not require active monetary intervention, just a floating exchange rate.

When oil production begins and the government starts spending the revenues, the terms of trade must appreciate a second time. Optimal policy prevents this happening though anticipatory inflation (as under a Peg) by tracking the natural interest rate and temporarily loosening policy in the quarter before the windfall arrives. This causes the nominal exchange rate to appreciate at exactly the time the windfall begins, via uncovered interest parity. Firms understand that the central bank will achieve the change in relative prices via the exchange rate (a single, flexible price), and thus have no need to change prices themselves (via many, sticky prices).

Welfare under optimal policy is higher than even the flexible-price outcome, shown in Table 4.1. This is because the central bank in an open economy can manipulate the terms of trade to benefit home consumers, which it does here by lowering the nominal interest rate further than the natural rate at $t = 16$, as discussed in Section 3.1. Some authors choose to neutralize this with an ad hoc employment subsidy that ensures the natural level of output stays constant (see

Gali and Monacelli, 2005). However we are interested in the way oil changes the natural level of output, so do not use a subsidy. It is also important to note that the effectiveness of loose policy in the future relies on firms understanding the monetary rule that is being followed. In the model this is taken care of using rational expectations. In practice it may require communication by the central bank about the nature of the rule.

4.1.4 Monetary Rules

To show that optimal policy can be easily implemented I now compare two Taylor rules. A simple Taylor rule that ignores how an oil discovery changes the natural rate will cause a recession between discovery and production. In contrast, a Taylor rule that properly accommodates these changes will replicate the flexible price outcome.

After discovering oil a simple Taylor rule will cause a recession between discovery and production. When oil is discovered domestic prices will immediately start rising, as forward looking firms anticipate future demand. The Taylor rule responds by tightening interest rates relative to where they would otherwise be. This exacerbates the contraction in output, causing a deep recession. The net effect is interest rates that are lower than they were before oil was discovered, but not low enough to avoid recession. The nominal exchange rate appreciates to compensate for lower interest rates as dictated by uncovered interest parity. The terms of trade also appreciates - not by firms raising prices, but rather by firms failing to lower prices as quickly as the nominal exchange rate appreciates. Welfare is higher than under a peg, because inflation is more stable, but lower than under an optimal rule (Table 4.1). Therefore, while the naive Taylor rule can perform well if shocks are contemporaneous, it performs poorly with news shocks as they alter the natural rate of interest.

A Taylor rule will replicate the flexible price outcome if it tracks the fall in the natural rate of interest before oil production begins, as in equation 3.5. The natural rate of interest falls because output is expected to rise, which lowers the returns to saving. Monetary policy that follows the natural rate will delay the second terms of trade adjustment until government spending begins. Firms realize this and so will not raise prices in anticipation, avoiding the negative output gaps that characterize the simple Taylor rule. Welfare under this rule replicates the flexible price case (Table 4.1).

Responding to the natural interest rate is increasingly seen as crucial for the appropriate conduct of monetary policy, and a Taylor rule that does so can approximate the flexible price

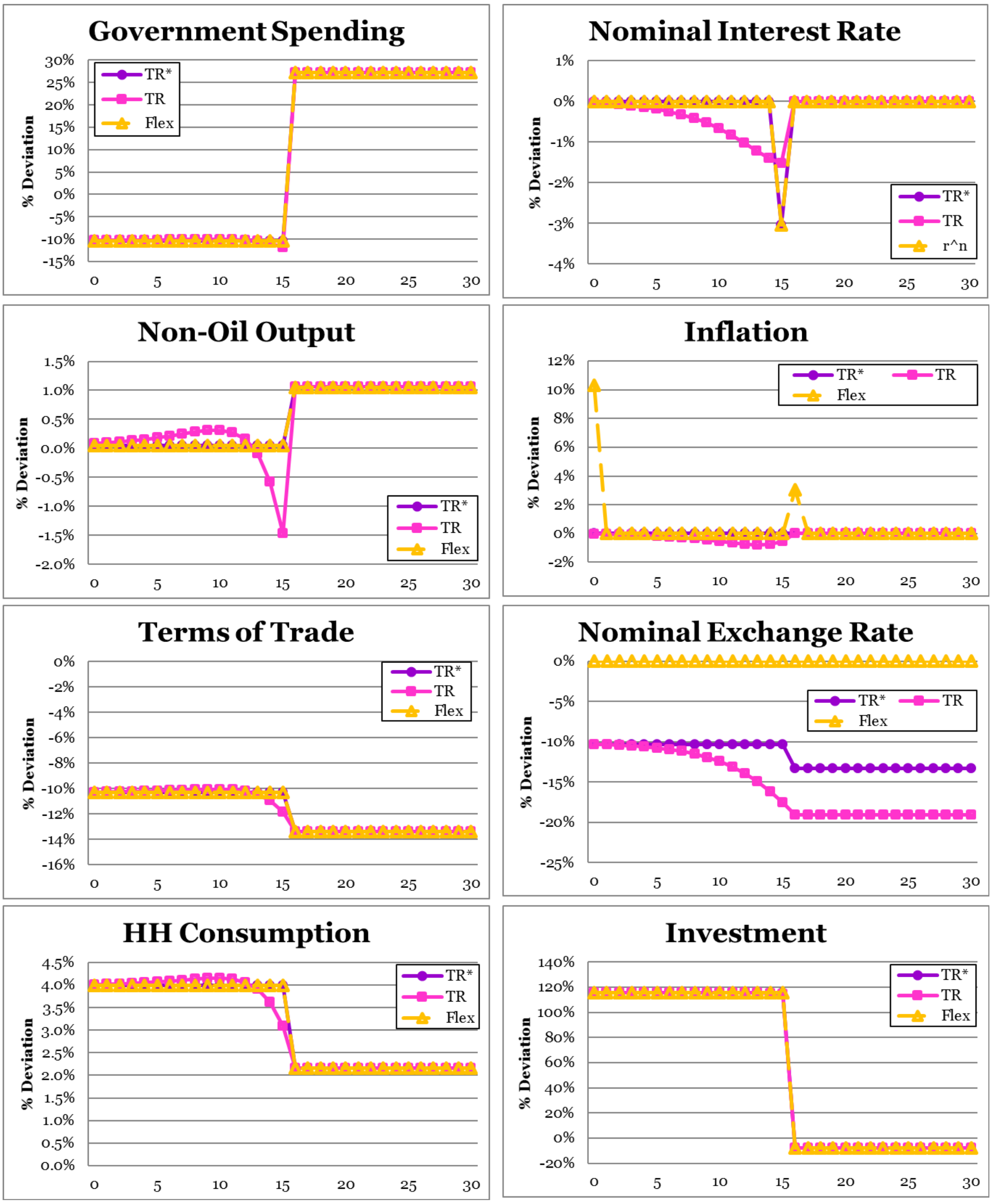

Figure 4.2: Forecast mean responses of key variables to an anticipated oil windfall, if the government spends oil revenues as they are received. Three monetary scenarios are illustrated: flexible prices (Flex), a simple Taylor rule (TR), and a correctly defined Taylor Rule (TR*). Horizontal axis shows time in quarters.

outcome (see Woodford, 2003; Laubach and Williams, 2003; and recent work by Barksy et al., 2014; and Curdia et al., 2015). However, this paper is the first to emphasize how news shocks will alter the natural rate of interest, by altering expectations of the natural level of output.

4.2 Bird-in-Hand Rule

A Norway-style Bird in Hand rule involves spending a fixed proportion of assets accumulated in a sovereign wealth fund each period. It is equivalent to setting the resource balance in Section 2.4 to $RB_t = \rho_{BH} F_t$, where foreign assets change according to, $E_t[M_{t,t+1}F_{t+1}] = (1 - \rho_{BH})F_t + P_{O,t}O_t$. In log-linear terms this means $\hat{g}_t = \hat{s}_t + \hat{\mathfrak{f}}_t$. Repeating the calibration in Appendix C under this rule sees government spending rise continuously as the sovereign wealth fund grows. The punchline is that an exchange rate peg or a simple Taylor rule perform poorly, but a Taylor rule that responds to changes in the natural rate of interest can replicate the flexible price outcome, as in the Spend-All rule.

4.2.1 Flexible Prices

To illustrate the two distinct phases of an oil discovery under a Bird-in-Hand rule I start with the case of flexible prices ("*Flex*" in Figure 4.3). The first phase begins at discovery ($t = 0$), when investment jumps. Investment requires domestic inputs, which causes the relative price of domestic goods (the terms of trade) to appreciate. During this phase foresighted households also immediately consume more on learning of their new-found wealth, appreciating the terms of trade further. Government consumption falls, as its budget constraint binds against the higher prices of domestic goods.

The second phase happens during production ($t \geq 16$). Domestic non-oil output may drop at the start of the boom, as oil investment ends. As this is anticipated, it causes the natural interest rate to jump up. Investment demand is then replaced by steadily rising government demand as oil wealth accumulates in the sovereign wealth fund. As this is also anticipated, the natural interest rate falls, in expectation of higher output in the future. The government only consumes home goods so their relative price (the terms of trade) appreciates further, causing domestic and foreign consumers to switch to foreign goods. Eventually oil production will end (not shown). Government spending will then decline as foreign assets are run down, causing the relative price of home goods to fall and consumption to rise to its new (wealthier) steady-state. Foresighted households will have chosen to borrow initially, before saving during the boom years, to sustain a permanently higher level of consumption once they end.

4.2.2 Exchange Rate Peg

Exchange rate pegs perform poorly if the government follows a Bird in Hand rule. This is because the continuous changes in government spending lead to sustained periods of inflation and deflation, and in turn large welfare losses. On discovering oil the terms of trade must appreciate. The nominal exchange rate is fixed, so this can only happen through an extended period of firms raising prices, both retrospectively and prospectively. Output will initially overshoot its natural level, as prices are slow to adjust, but will quickly fall as firms raise prices in anticipation of government demand. Once oil production begins, government spending will rise for the life of the field. The price of home goods must also rise but it will consistently be below, and output above, its natural level due to nominal rigidities. Eventually, when the end of the boom approaches (not shown), forward-looking firms will stop raising their prices as they anticipate the decline of government demand. This causes output to rise even further beyond its natural rate, just as the boom comes to an end. After oil production ends, government spending will decline until the sovereign wealth fund is depleted. In the inverse of the accumulation phase, the price of domestic goods will fall too slowly and so depress consumption and output below their natural levels (a negative output gap).

4.2.3 Naive Taylor Rule

A naive Taylor rule that ignores changes in the natural rate of interest ($\phi_\pi = 1.5$, $\phi_y = 0.5$) will lead to an extended period of negative output gaps while oil output is rising, and positive output gaps when oil output is falling. This is because firms are forward-looking, and so will be raising their prices during the output phase, and lowering prices during the decline phase. In response the central bank will progressively tighten policy against forward-looking inflation (and loosen against forward-looking deflation). This forces output away from its natural level, as the policy rate fails to track the natural rate of interest.

4.2.4 Optimal Taylor Rule

A Taylor rule that follows the natural interest rate will avoid the output gap fluctuations associated with the Bird in Hand rule. As in Section 4.1 this rule lets the nominal exchange rate appreciate when oil is discovered, allowing relative prices to adjust to extra forward-looking

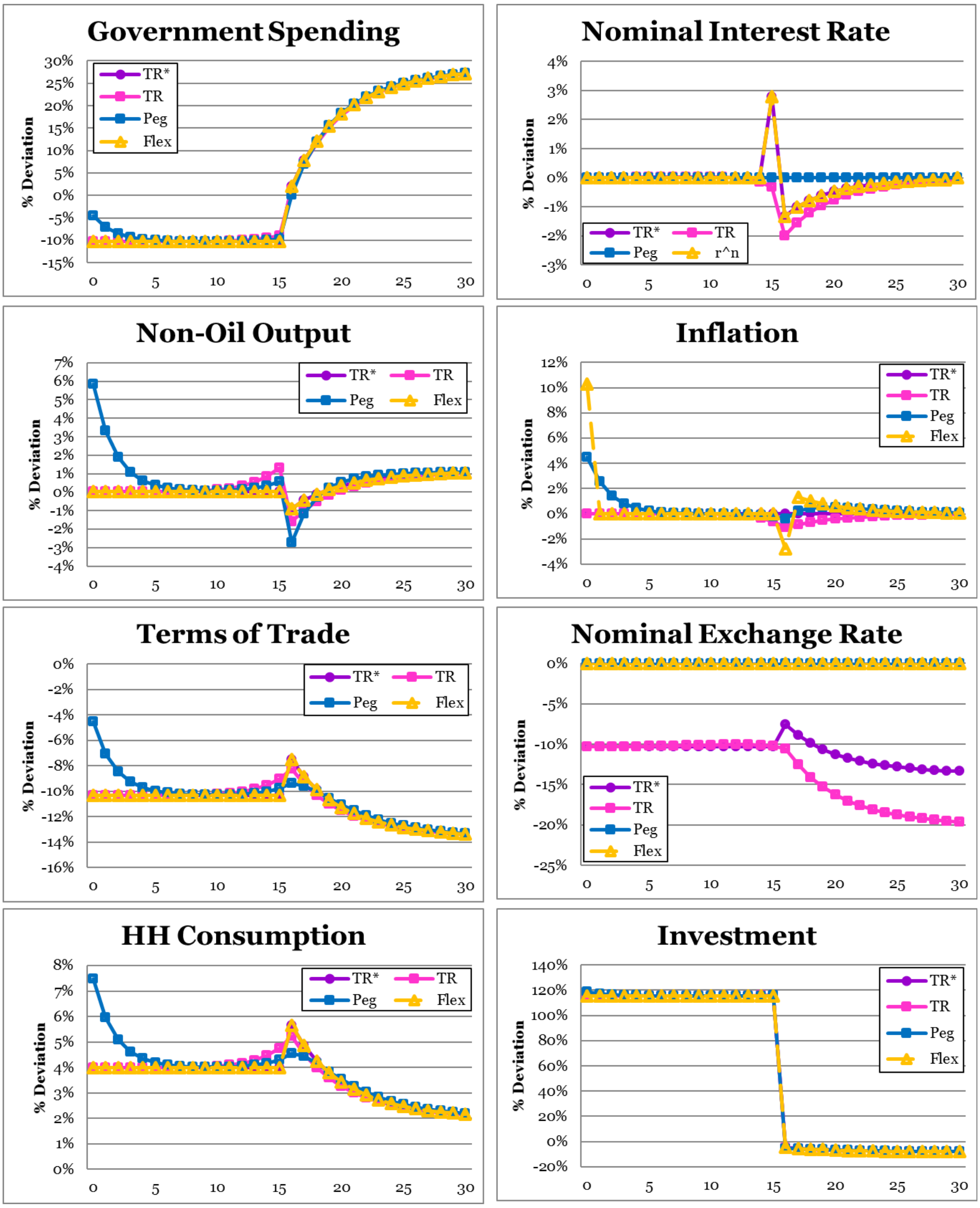

Figure 4.3: Forecast mean responses of key variables to an anticipated oil windfall under the Bird in Hand rule. Four different scenarios are illustrated: flexible prices (Flex), a currency peg (Peg), a simple Taylor rule (TR), and a correctly defined Taylor Rule (TR*). Horizontal axis shows time in quarters.

consumption and investment. Policy should tighten just before the end of the investment boom, preventing forward-looking firms from lowering their prices, and loosen as government spending begins, preventing firms from raising their prices prospectively. During oil production monetary policy will slowly and continually tighten, to offset low sticky prices and prevent output from overshooting its natural level. Just before production ends policy should tighten further to delay the terms of trade appreciation until it is necessary. As spending declines monetary policy should gradually loosen to offset sub-optimally high prices.

5 Conclusion

This paper considers how monetary policy should optimally respond to news of an oil discovery. Discovering oil provides news that the natural level of output will be higher in the future, which lowers the natural rate of interest. If the central bank does not let the policy rate track the natural rate of interest, like under an exchange rate peg or a naive Taylor rule, then it will lead to inflation and a recession. However, these issues can be overcome if the Taylor rule properly responds to changes in the natural rate.

To make this point the paper extends a standard small open economy model in two ways: allowing for oil revenues and news shocks. The model is parsimonious enough to permit an explicit derivation of the micro-founded central bank loss function, and in turn optimal policy. The model is then used to study how an economy would respond to a stylized oil discovery under two cases: if all oil revenues are spent as they are received, or saved in a sovereign wealth fund and spent following a rule like Norway's (see Appendix B.4). In both cases optimal policy is compared to an exchange rate peg (used by 75% of resource-dependent economies), and Taylor rules that either do, or do not, respond to changes in the natural interest rate.

The paper provides one of the first studies of optimal monetary policy for oil exporters, and the first to study how monetary policy should respond to an oil discovery. The paper also provides one of the first studies of the optimal monetary response to a news shock, and the first on news about future demand - like an oil shock. In doing so it emphasizes that there is an intimate link between news shocks and the natural rate of interest, which is crucial for the conduct of monetary policy.

Extensions to this work may consider other price-setting assumptions, and include more sectors in the economy. Price-setting plays an important role in this model, because forward-looking firms raising their prices before an oil boom causes a recession under some monetary regimes. Alternative, state-based price-setting behaviour would be an interesting extension (such as Gertler and Leahy, 2008 and Golosov and Lucas, 2007). If there are concave menu costs then there may be an incentive to delay price rises as long as possible, overcoming distortions caused by firms raising prices in advance. Adding other sectors, like a non-traded sector, would give a useful insight into monetary policy's role in managing Dutch disease.

While this analysis was conducted with an oil discovery in mind, it may also be of interest to other applications. Aid windfalls have similar characteristics to oil windfalls. If anticipating a windfall leads to recession, then it can be argued that aid grants should only be announced when government spending begins. Anticipated increases in government demand may also happen between an election and the implementation of a budget, though this would require a closer investigation of taxation.

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A Appendix: Model

A.1 International Risk Sharing

To derive the expressions in equations 2.9 and 2.10 we begin with the household budget constraint in equation 2.2. Summing this constraint over an infinite horizon, and using the transversality condition, $\lim_{T\to\infty} M_{0,T}D_T = 0$, gives $E_t[\sum_{s=0}^{\infty} M_{t,t+s}P_{t+s}C_{t+s}] = E_t[\sum_{s=0}^{\infty} M_{t,t+s}(W_{t+s}N_{t+s} - P_{H,t+s}T_{t+s})] + D_t$. Combining this with the Euler equation 2.4 gives,

$$P_tC_t \;\;=\;\; E_t\left[\sum_{s=0}^{\infty} M_{t,t+s}\left(W_{t+s}N_{t+s} - P_{H,t+s}T_{t+s}\right) + D_t\right] / E_t\left[\sum_{s=0}^{\infty}\beta^s\right] \tag{A.1}$$

Combining this with a similar condition that holds in each country, f, and making use of the international risk-sharing condition in equation 2.7 gives the results in 2.9 and 2.10.

To express relative household wealth in terms of government spending, as in equation 2.12, we start with its expression at time $t = 0$ assuming $D_0 = D_f = 0$,

$$\Theta_{f,0} = \frac{E_0[\sum_{t=0}^{\infty} M_{0,t}(W_tN_t - P_{H,t}T_t)]}{E_0[\sum_{t=0}^{\infty} M_{0,t}^f(W_t^fN_t^f - P_{f,t}^fT_t^f)]}$$

If prices are flexible, and if the government provides a small wage subsidy to correct for monopolistic distortions, then $W_t = P_{H,t}\forall t$. Using the small open economy assumption all foreign variables remain at their steady state, and $r^* = \rho$. Thus, the denominator becomes, $E_0[\sum_0^{\infty} M_{0,t}^f(W_t^fN_t^f - P_{f,t}^fT_t^f)] = P_f^s(N_f^s - T_f^s)(1+\rho)/\rho$. In the steady state we assume that taxes are the same as the rest of the world, so $\gamma_T = \chi$. Therefore, $(N^s - T^s) = (N_f^s - T_f^s) = 1 - \chi$

and $P_H^s/P_f^s = (S_f^s)^{-1} = \Theta_f^s$. Using this, and, $\beta = (1+\rho)^{-1}$, we can log-linearize the definition of $\Theta_{f,0}$ around the steady state assuming prices are flexible,

$$\begin{aligned}
\hat{\vartheta}_{f,0}^n \Theta_f^s &= \tfrac{P_H^s(N^s - T^s)}{P_f^s(N_f^s - T_f^s)(1+\rho)/\rho} E_0 \left[\sum_{t=0}^{\infty} M_{0,t}(\hat{p}_{H,t}^n + \tfrac{1}{1-T^s/N^s}(\hat{n}_t^n - \tfrac{T^s}{N^s}\hat{\mathfrak{t}}_t^n)) \right] \\
\hat{\vartheta}_{f,0}^n &= \tfrac{\rho}{1+\rho} E_0 \left[\sum_{t=0}^{\infty} M_{0,t}(\hat{p}_{H,t}^n + \tfrac{1}{1-\chi}(\hat{n}_t^n - \chi \hat{\mathfrak{t}}_t^n)) \right]
\end{aligned}$$

Note that taxes are $\hat{\mathfrak{t}}_t = \ln T_t - \ln T^s$, rather than $\hat{t}_t = -\ln(1 - \frac{T_t}{G_t}) + \ln(1 - \frac{T^s}{G^s})$ which we use in the text. We assume that $\hat{t}_t = 0$, so $\hat{\mathfrak{t}}_t = \hat{g}_t$. Under flexible prices it is necessary to fix a numeraire, so we let $\hat{e}_t = 0$ such that $\hat{s}_t^n = -\hat{p}_{H,t}^n$. Also, combining equations 2.19 and A.4 gives $\varphi \hat{y}_t^n = -\hat{s}_t^n - \hat{\vartheta}_t^n$. So, $\hat{p}_{H,t}^n = \varphi \hat{y}_t^n + \hat{\vartheta}_t^n$ and $\hat{g}_t^n = -\varphi \hat{y}_t^n - \hat{\vartheta}_t^n + \widehat{rb}_t$. Therefore,

$$\begin{aligned}
\hat{p}_{H,t} + \tfrac{1}{1-\chi}(\hat{n}_t - \chi \hat{\mathfrak{t}}_t) &= \varphi \hat{y}_t^n + \hat{\vartheta}_t^n + \tfrac{1}{1-\chi}(\hat{y}_t - \chi(-\varphi \hat{y}_t^n - \hat{\vartheta}_t^n + \widehat{rb}_t)) \\
&= \tfrac{1+\varphi}{1-\chi} \hat{y}_t^n + \tfrac{1}{1-\chi} \hat{\vartheta}_t^n - \tfrac{\chi}{1-\chi} \widehat{rb}_t \\
&= \tfrac{1+\varphi}{1-\chi} \left\{ \tfrac{\gamma_G}{(1+\varphi)} \widehat{rb}_t - \tfrac{(\alpha + \gamma_G(1-\alpha))}{(1+\varphi)} \hat{\vartheta}_t^n \right\} + \tfrac{1}{1-\chi} \hat{\vartheta}_t^n - \tfrac{\chi}{1-\chi} \widehat{rb}_t \\
&= \tfrac{\gamma_G - \chi}{1-\chi} \widehat{rb}_t + \tfrac{(1-\gamma_G)(1-\alpha)}{1-\chi} \hat{\vartheta}_t^n
\end{aligned}$$

Substituting this into the above relationship, and using effective (rather than bilateral) relationships gives the following, where we make use of the observation that $E_t[\hat{\vartheta}_{t+s}^n] = \hat{\vartheta}_t^n \forall s$, as relative wealth incorporates all expected future income,

$$\begin{aligned}
\hat{\vartheta}_0^n &= \tfrac{\rho}{1+\rho} E_0 \left[\sum_{t=0}^{\infty} M_{0,t}(\tfrac{\gamma_G - \chi}{1-\chi} \widehat{rb}_t + \tfrac{(1-\gamma_G)(1-\alpha)}{1-\chi} \hat{\vartheta}_t^n) \right] \\
&= \tfrac{\rho}{1+\rho} E_0 \left[\sum_{j=0}^{\infty} M_{0,t} \tfrac{\gamma_G - \chi}{1-\chi} \widehat{rb}_t \right] + \tfrac{\rho}{1+\rho} \tfrac{(1-\gamma_G)(1-\alpha)}{1-\chi} \tfrac{1+\rho}{\rho} \hat{\vartheta}_0^n \\
&= \tfrac{(1-\beta)(\gamma_G - \chi)}{(1-\chi) - (1-\gamma_G)(1-\alpha)} E_0 \left[\sum_{j=0}^{\infty} M_{0,t} \widehat{rb}_t \right]
\end{aligned}$$

Assuming households calculate their lifetime wealth under flexible prices (so the problem is much more tractable and short-term nominal rigidities do not have permanent effects), $\hat{\vartheta}_t = \hat{\vartheta}_t^n$, gives the relationship in equation 2.12. It is important to note that this is an approximation, which is subject to two types of error. The first is that the goods market equilibrium (equation A.2) is not log-linear, and so may require a higher order approximation. The second is that we approximate $-\alpha \hat{\vartheta}_t \approx \ln\left[(1-\alpha) + \alpha \Theta_t^{-1}\right] - \ln\left[(1-\alpha) + \alpha (\Theta^s)^{-1}\right]$ when approximating the goods market equilibrium. These errors can be reduced by making the jump and the distance from $\Theta^s = 1$ small.

A.2 Aggregate Demand

This appendix derives a first-order approximation of aggregate demand (the IS curve). The market clearing condition for each domestically produced good i, at each time t, requires output to equal demand from domestic consumption, investment, government purchases and consumption from each foreign country f,

$$\begin{aligned} Y_t(i) &= C_{H,t}(i) + \int_0^1 C^f_{H,t}(i) df + J_{H,t}(i) + G_t(i) \\ &= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} \left\{(1-\alpha)\left(\frac{P_t C_t}{P_{H,t}}\right) + \alpha \int_0^1 \left(\frac{\varepsilon_{f,t} P^f_t C^f_t}{P_{H,t}}\right) df + (1-\alpha)\left(\frac{P_t J_t}{P_{H,t}}\right) + G_t\right\} \end{aligned}$$

The second equality above assumes symmetric preferences across countries implying, $C^f_{H,t}(i) = \alpha\left(P_{H,t}(i)/P_{H,t}\right)^{-\epsilon}\left(\varepsilon_{f,t} P^f_t / P_{H,t}\right) C^f_t$. Aggregating across goods and using 2.15 gives,

$$\begin{aligned} Y_t &= (1-\alpha)\left(\frac{P_t C_t}{P_{H,t}}\right) + \alpha \int_0^1 \left(\frac{\varepsilon_{f,t} P^f_t C^f_t}{P_{H,t}}\right) df + (1-\alpha)\left(\frac{P_t J_t}{P_{H,t}}\right) + G_t \\ &= S^\alpha_t \left[(1-\alpha) C_t + \alpha \int_0^1 \mathcal{Q}_{f,t} C^f_t df + (1-\alpha)\Theta_{J,t}\Theta_t^{-1} C_t\right] + G_t \\ &= C_t S^\alpha_t \left[(1-\alpha) + \alpha\Theta_t^{-1} + (1-\alpha)\Theta_{J,t}\Theta_t^{-1}\right] + G_t \end{aligned} \tag{A.2}$$

Log-linearising this to the first order gives,

$$\hat{y}_t = (1-\gamma_G)\left(\hat{c}_t + \alpha\hat{s}_t - \alpha\hat{\vartheta}_t + (1-\alpha)\hat{\vartheta}_{J,t}\right) + \gamma_G \hat{g}_t \tag{A.3}$$

where $\gamma_G \equiv G^s/Y^s$, $-\alpha\hat{\vartheta}_t \approx \ln\left[(1-\alpha) + \alpha\Theta_t^{-1}\right] - \ln\left[(1-\alpha) + \alpha(\Theta^s)^{-1}\right]$ and $(1-\alpha)\hat{\vartheta}_{J,t} \approx ln[(1-\alpha)\Theta_{J,t}\Theta_t^{-1}] - ln[(1-\alpha)\Theta^s_{J,t}\Theta_t^{s-1}]$ when $\Theta^s \approx 1$. This approximation makes the analysis far more tractable without a major loss in accuracy, as discussed in Appendix D. Combining this with 2.11 and $\hat{g}_t = \hat{s}_t + \widehat{rb}_t$ gives an expression for output and the terms of trade,

$$\hat{y}_t = \hat{s}_t + \gamma_G \widehat{rb}_t + (1-\gamma_G)(\hat{c}^*_t + (1-\alpha)(\hat{\vartheta}_t + \hat{\vartheta}_{J,t})) \tag{A.4}$$

$$\hat{s}_t = \tfrac{1}{1-\gamma_G}\hat{y}_t - \tfrac{\gamma_G}{1-\gamma_G}\hat{g}_t - \hat{c}^*_t - (1-\alpha)\hat{\vartheta}_t - (1-\alpha)\hat{\vartheta}_{J,t} \tag{A.5}$$

The IS curve in 2.17 can be found by combining 2.4; $\pi_t = \pi_{H,t} + \alpha\Delta s_t$; $\tilde{c}_t = (1-\alpha)\tilde{s}_t$ from 2.11; $\tilde{y}_t = \tilde{s}_t$ from A.4; and $\Delta c_t^n + \alpha\Delta s_t^n = -\varphi\Delta n_t^n + \Delta a_t$ from 2.3 and $mc_t^n = -\nu + w_t^n - p_{H,t}^n - a_t = -\mu$ to give, $\tilde{y}_t = E_t[\tilde{y}_{t+1}] - (i_t - E_t[\pi_{H,t+1}] - r_t^n)$, where the natural rate of interest is, $r_t^n = \rho + (1+\varphi)\Delta a_{t+1}^n - \varphi E_t[\Delta\hat{y}_{t+1}^n]$. This can be rearranged using A.4 and 2.19 to give $\hat{r}_t^n = \rho + E_t[\Delta\hat{s}_{t+1}^n] + E_t[\Delta\hat{\vartheta}_{t+1}^n] + E_t[\Delta\hat{c}_{t+1}^*]$.

A.3 Price-Setting and Aggregate Supply

This appendix derives the optimal price-setting rule for firms, and first- and second-order approximations of aggregate supply. The first-order approximation is the New Keynesian Phillips Curve, and the second-order one is used to incorporate higher order effects in the central bank's loss function (see Gali, 2008 Ch 3; Gali and Monacelli, 2005; and Benigno and Woodford, 2005).

Each period a measure of $(1-\theta)$ randomly selected firms choose the price, $P_{H,t}^*$ that maximizes profits according to Calvo (1983), $\max_{P_{H,t}^*} \sum_{k=0}^{\infty} \theta^k E_t[M_{t,t+k}(P_{H,t}^* Y_{t+k} - \Psi_{t+k}(Y_{t+k}))]$, where $\Psi_{t+k}(\cdot)$ is the nominal cost function. The first-order condition to this problem is, $\sum_{k=0}^{\infty} \theta^k E_t[M_{t,t+k} Y_{t+k}(P_{H,t}^* - \frac{\epsilon}{\epsilon-1}\psi_{t+k})] = 0$, where $\psi_t = P_{H,t} MC_t$ is the nominal marginal cost. Dividing throughout by the price level in the previous period gives, $\sum_{k=0}^{\infty} \theta^k E_t[Q_{t,t+k} Y_{t+k}(P_{H,t}^*/P_{H,t-1} - \frac{\epsilon}{\epsilon-1} MC_{t+k}\Pi_{H,t-1,t+k})] = 0$, where $\Pi_{H,t-1,t+k} = P_{H,t+k}/P_{H,t-1}$ is gross inflation between time $t-1$ and $t+k$. Now, we take a second-order log-linearisation of this condition around the zero-inflation steady state. Note that this is the economy's state in the absence of an oil discovery, so inflation is zero and output is Y^s; rather than the flexible price state once oil is discovered, Y_t^n. This gives the following relationship for every k,

$$\begin{aligned}
\theta^k E_t[Q_{t,t+k} Y_{t+k}(\frac{P_{H,t}^*}{P_{H,t-1}} - \tfrac{\epsilon}{\epsilon-1} MC_{t+k}\Pi_{H,t-1,t+k})] \approx \; & \theta^k \beta^k Y E_t[(p_{H,t}^* - p_{H,t-1}) + \tfrac{1}{2}(p_{H,t}^* - p_{H,t-1})^2 \\
& -\widehat{mc}_{t+k} - \tfrac{1}{2}\widehat{mc}_{t+k}^2 \\
& -(p_{H,t+k} - p_{H,t-1}) - \tfrac{1}{2}(p_{H,t+k} - p_{H,t-1})^2 \\
& -\widehat{mc}_{t+k}(p_{H,t+k} - p_{H,t-1})] + o(\widehat{mc}_{t+k}^3)
\end{aligned}$$

Summing over all periods and using $\pi_{H,t} = (1-\theta)(p_{H,t}^* - p_{H,t-1})$ gives,

$$\begin{aligned}
\tfrac{1}{1-\theta}\pi_{H,t} + \tfrac{1}{2}(\tfrac{1}{1-\theta}\pi_{H,t})^2 = \; & (1-\theta\beta)\sum_{k=0}^{\infty} \theta^k \beta^k E_t[(p_{H,t+k} - p_{H,t-1}) + \tfrac{1}{2}(p_{H,t+k} - p_{H,t-1})^2 \\
& +\widehat{mc}_{t+k} + \tfrac{1}{2}\widehat{mc}_{t+k}^2 + \widehat{mc}_{t+k}(p_{H,t+k} - p_{H,t-1})] + o(\widehat{mc}_{t+k}^3) \quad \text{(A.6)}
\end{aligned}$$

Rearranging this expression and dropping terms of order two and above gives the result in equation 2.16. To derive a second-order approximation of aggregate supply we make use of the recursive relationship in the following lemma,

Lemma 1. *Equation A.6 can be expressed recursively as,*

$$V_t \;=\; \lambda[\widehat{mc}_t + \tfrac{1}{2}\widehat{mc}_t^2 + \tfrac{1}{2}c_\pi \pi_{H,t}^2] + \beta E_t[V_{t+1}] + t.i.p. + o(\widehat{mc}_t^3) \tag{A.7}$$

where, $V_t = \pi_{H,t} + \frac{1}{2}\frac{1}{\theta(1-\theta)}\pi_{H,t}^2$, $\lambda = \frac{(1-\theta\beta)(1-\theta)}{\theta}$, *and* $c_\pi = \frac{1+\theta}{\theta\lambda}$.

Proof. The proof relies on rearranging the order of summation of the infinite sums in equation A.6, similar to Benigno and Woodford (2005). We begin with,

$$\begin{aligned}
\sum_{k=0}^{\infty}\theta^k\beta^k E_t[(p_{H,t+k} - p_{H,t-1})] &= \sum_{k=0}^{\infty}\theta^k\beta^k E_t[\sum_{s=0}^{k}\pi_{H,t+s}] \\
&= \frac{1}{1-\theta\beta}\sum_{k=0}^{\infty}\theta^k\beta^k E_t[\pi_{H,t+k}] \\
&= \frac{1}{1-\theta\beta}(\pi_{H,t} + P_t), \qquad\qquad (A.8)
\end{aligned}$$

where P_t is defined as the sum of all future inflation, $P_t \equiv \sum_{k=1}^{\infty}\theta^k\beta^k E_t[\pi_{H,t+k}]$. Next we have,

$$\begin{aligned}
\sum_{k=0}^{\infty}\theta^k\beta^k E_t[(p_{H,t+k} - p_{H,t-1})^2] &= \sum_{k=0}^{\infty}\theta^k\beta^k E_t[(\sum_{s=0}^{k}\pi_{H,t+s})^2] \\
&= \tfrac{1}{1-\theta\beta}\sum_{k=0}^{\infty}\theta^k\beta^k E_t[\pi_{H,t+k}(\pi_{H,t+k} + 2P_{t+k})], \qquad (A.9)
\end{aligned}$$

where we make use of a quadratic expansion and rearrange the order of summation. Third we have,

$$\begin{aligned}
\sum_{k=0}^{\infty}\theta^k\beta^k E_t[\widehat{mc}_{t+k}(p_{H,t+k} - p_{H,t-1})] &= \sum_{k=0}^{\infty}\theta^k\beta^k E_t[\widehat{mc}_{t+k}(\sum_{s=0}^{k}\pi_{H,t+s})] \\
&= \sum_{k=0}^{\infty}\theta^k\beta^k E_t[\pi_{H,t+k}N_{t+k}]. \qquad\qquad (A.10)
\end{aligned}$$

where N_t is defined as the sum of all future deviations of marginal costs from the steady state, $N_t \equiv \sum_{s=0}^{\infty}\theta^s\beta^s\widehat{mc}_{t+s}$. To find a recursive expression for equation A.6 substitute in A.8, A.9

and A.10 to give,

$$\begin{aligned}
\tfrac{1}{1-\theta}\pi_{H,t} + \tfrac{1}{2}(\tfrac{1}{1-\theta}\pi_{H,t})^2 &= (1-\theta\beta)\sum_{k=0}^{\infty}\theta^k\beta^k E_t[\tfrac{1}{1-\theta\beta}\pi_{H,t+k} + \widehat{mc}_{t+k} + \tfrac{1}{2}\widehat{mc}_{t+k}^2 \\
&\quad + \tfrac{1}{2}\tfrac{1}{1-\theta\beta}\Big(\pi_{H,t+k}\{\pi_{H,t+k} + 2P_{t+k}\}\Big) + \pi_{H,t+k}N_{t+k}] + o(\widehat{mc}_{t+k}^3).
\end{aligned}$$

This can be expressed recursively as,

$$\begin{aligned}
\tfrac{1}{1-\theta}\pi_{H,t} + \tfrac{1}{2}(\tfrac{1}{1-\theta}\pi_{H,t})^2 &= (1-\theta\beta)\Big(\tfrac{1}{1-\theta\beta}\pi_{H,t+k} + \widehat{mc}_t + \tfrac{1}{2}\widehat{mc}_t^2 \\
&\quad + \tfrac{1}{2}\tfrac{1}{1-\theta\beta}\pi_{H,t}\{\pi_{H,t} + 2P_t\} + \pi_{H,t}N_t\Big) \\
&\quad + \theta\beta E_t[\tfrac{1}{1-\theta}\pi_{H,t+1} + \tfrac{1}{2}(\tfrac{1}{1-\theta}\pi_{H,t+1})^2] \qquad \text{(A.11)}
\end{aligned}$$

To eliminate $N_t = \sum_{s=0}^{\infty}\theta^s\beta^s\widehat{mc}_{t+s}$ we note that we will only need an expression for N_t that is accurate to the first order, as it will be multiplied by $\pi_{H,t}$ in the above expression. To the first order equation A.6 is,

$$\begin{aligned}
\tfrac{1}{1-\theta}\pi_{H,t} &\approx (1-\theta\beta)\sum_{k=0}^{\infty}\theta^k\beta^k E_t[(p_{H,t+k} - p_{H,t-1}) + \widehat{mc}_{t+k}] + o(\widehat{mc}_{t+k}^2) \\
&= (1-\theta\beta)\left\{\frac{1}{1-\theta\beta}(\pi_{H,t} + P_t) + N_t\right\} + o(\widehat{mc}_{t+k}^2) \\
(1-\theta\beta)N_t &= \tfrac{\theta}{1-\theta}\pi_{H,t} - P_t
\end{aligned}$$

Substituting this into A.11, collecting terms and multiplying by $(1-\theta)/\theta$ gives the expression in A.7. □

First-Order Approximation: The New Keynesian Phillips Curve To a first-order approximation equation A.7 gives the standard New Keynesian Phillips curve,

$$\pi_{H,t} \approx \beta E_t[\pi_{H,t+1}] + \lambda\widehat{mc}_t \qquad \text{(A.12)}$$

where $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$ and $\widehat{mc}_t$ is the deviation of real marginal costs in time t from their steady state level. Real marginal costs are common across domestic firms. If prices are flexible them real marginal costs are, $mc_t^n = -\mu$, which is the flexible price limit of 2.16. If prices are sticky then real marginal costs are, $mc_t = -\nu + w_t - p_{H,t} - a_t$, where $\nu \equiv -\ln(1-\tau)$ and τ is an employment subsidy that offsets the marginal cost distortion of monopolistic competition. The subsidy means that $mc_t^n - mc^s = -\mu + \nu = 0$, so $\widehat{mc}_t = \widetilde{mc}_t$.

Now we will express the Phillips curve in terms of the aggregate output gap. Aggregate domestic output is given by the index, $Y_t \equiv \left[\int_0^1 Y_t(i)^{(\epsilon-1)/\epsilon}\right]^{\epsilon/(\epsilon-1)}$, similar to consumption. Aggregate employment is given by $N_t \equiv \int_0^1 N_t(i)di = \frac{Y_t Z_t}{A_t}$, where $Z_t \equiv \int_0^1 \frac{Y_t(i)}{Y_t}di = \int_0^1 \left(\frac{P_{H,,t}(i)}{P_{H,t}}\right)^{-\epsilon}$ is a measure of price dispersion. In Appendix B.1 we follow Gali and Monacelli (2008) and show that equilibrium variations in $z_t \equiv \ln Z_t$ around the perfect foresight steady state are of second order. So, up to a first-order approximation aggregate output is, $y_t = a_t + n_t$.

The deviation of real marginal costs from the steady state can be expressed as a function of domestic output, $\widehat{mc}_t = (\frac{1}{1-\gamma_G} + \varphi)\hat{y}_t - \frac{\gamma_G}{1-\gamma_G}\hat{g}_t + \alpha\hat{\vartheta}_t - (1-\alpha)\hat{\vartheta}_{J,t} - (1+\varphi)\hat{a}_t$, using 2.3, the goods market equilibrium A.3, and $y_t = a_t + n_t$. Therefore $\widetilde{mc}_t = \widehat{mc}_t - \widehat{mc}_t^n = (\frac{1}{1-\gamma_G} + \varphi)\tilde{y}_t - \frac{\gamma_G}{1-\gamma_G}\tilde{g}_t$ as $\tilde{\vartheta}_t = \tilde{\vartheta}_{J,t} = \tilde{a}_t = 0$. Note that $\hat{g}_t \neq \hat{g}_t^n$ because oil revenue is received in foreign currency, so the government's purchasing power is affected by the nominal exchange rate if prices are sticky. Expressing government spending in terms of the resource balance gives, $\hat{g}_t = \hat{s}_t + \widehat{rb}_t$. Combining this with equations 2.15, A.4 and $\widehat{rb}_t = \widehat{rb}_t^n$ gives $\tilde{g}_t = \tilde{s}_t = \tilde{y}_t$. Therefore, $\widehat{mc}_t = \widetilde{mc}_t = (1+\varphi)\tilde{y}_t$ and we have the standard New Keynesian Phillips Curve in equation 2.20, $\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \lambda(1+\varphi)(\hat{y}_t - \hat{y}_t^n)$, keeping track of both the actual and natural levels of output, $\tilde{y}_t \equiv \hat{y}_t - \hat{y}_t^n$. The expression for $\hat{y}_t^n$ in 2.19 can be derived using $\widehat{mc}_t^n = 0$, $\hat{g}_t^n = \hat{s}_t^n + \widehat{rb}_t$ and equations 2.15 and A.4.

Second-Order Approximation Iteratively combining A.7 gives the following, which we will later substitute into the central bank loss function,

$$V_0 = E_0 \sum_{t=0}^{\infty} \beta^{t-t0} \lambda \left\{ \widehat{mc}_t + \tfrac{1}{2}\widehat{mc}_t^2 + \tfrac{1}{2}c_\pi \pi_{H,t}^2 \right\} + t.i.p. + o(\widehat{mc}_t^3) \tag{A.13}$$

Now, we have $\widehat{mc}_t = \widetilde{mc}_t + \widehat{mc}_t^n = \widetilde{mc}_t = (1+\varphi)\tilde{y}_t$ as real marginal costs when prices are flexible will be the same as in the steady state, $\widehat{mc}_t^n = 0$. which is accurate to the second order.

A.4 The Steady State

A.4.1 Symmetric case without oil

When the home country receives no resource revenues then the steady state will be symmetric at home and abroad, and defined from the perspective of a benevolent social planner. This is both a benchmark and a way to define the steady state abroad. Symmetry implies $\Theta = S = 1$.

The social planner solves the following problem,

$$\max_{C_i^i, C_f^i, G^i, N^i} \mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ [(1-\chi)\ln C^i + \chi_t \ln G^i - \frac{N^{i(1+\varphi)}}{1+\varphi}]$$
$$-\lambda_1 [A^i N^i - G^i - C_i^i - \int_0^1 C_i^f df] - \lambda_2 [(C_i^i)^{1-\alpha} (C_f^i)^{\alpha} \tfrac{1}{(1-\alpha)\alpha} - C^i]$$

The solution is given by, $N^i = 1$, $Y^i = A^i$,$C^i = (1-\chi)(A^i)^{1-\alpha}(\int_0^1 A^f df)^{\alpha}$, $C_i^i = (1-\alpha)(1-\chi)A^i$, $C_i^f = \alpha(1-\chi)A^i$ and $G^i = \chi A^i$. Steady state output is not affected by the share of government spending, χ, or the degree of openness, α, because of symmetry. A change in either would affect both domestic and foreign demand for domestic production, which will offset one another. The optimal allocation for the world as a whole can be found by setting $\alpha \to 0$.

A.4.2 Asymmetric case with oil

When the home government receives oil income the steady state will depend on fiscal policy, and will no longer be symmetric with the rest of the world. It is found by simultaneously solving five equations: $(1-\chi)AN^{-\varphi} = S^{\alpha}C$, $AN - T = S^{\alpha}C$, $AN = CS^{\alpha}\left[(1-\alpha) + \alpha\Theta^{-1} + (1-\alpha)\Theta_J\Theta^{-1}\right] + G$, $C = S^{1-\alpha}\Theta C^*$, $J = S^{1-\alpha}\Theta_J C^*$; using $T = \gamma_T AN$, $J = \gamma_J AN$, $G = \gamma_G AN$, and $C^* = (1-\chi)A$ where $A = A^*$. The first equation comes from combining 2.3 and the steady state marginal cost condition $MC^s = W/(P_H A) = 1$. The second follows from the household budget constraint, 2.2, assuming there is no permanent accumulation of foreign assets $D^s = 0$. The third is the goods market equilibrium, A.2. The fourth and fifth are the international risk sharing conditions, 2.11 and 2.15. Solving simultaneously gives the results in equation 2.21.

The steady state in 2.21 will collapse to the symmetric case under certain fiscal policies. If taxes are chosen optimally, $\gamma_T = \chi$, then output will be the same as in the symmetric case, $N^s = 1$. If the government receives no oil revenues, $\gamma_G = \gamma_T$, then we have the symmetric case with $\Theta^s = S^s = 1$. We will proceed assuming $\gamma_T = \chi$ for tractability, and $\gamma_G > \gamma_T$. We will define the change in household wealth on discovering oil by comparing Θ in the steady states before and after discovery.

B Appendix: Monetary Policy

B.1 Central Bank Loss Function

This appendix derives a second-order approximation of the central bank's loss function. This allows us to study optimal policy under commitment from a timeless perspective, as in Benigno and Woodford (2005). We begin with the household utility function 2.1, $U_0 = E_0 \sum_t^\infty \beta^t \{(1-\chi)\ln C_t + \chi \ln G_t - \frac{N_t^{1+\varphi}}{1+\varphi}\}$. Taking a second-order Taylor expansion of $\ln(Y_t - G_t)$ around the steady state, using, $\gamma_G = G/Y$, $(Y_t - Y)/Y \approx \hat{y}_t + \frac{1}{2}\hat{y}_t^2$ and collecting terms independent of policy $(t.i.p)$ gives,

$$\begin{aligned}
\ln(Y_t - G_t) &\approx \ln(Y_s - G_s) + \tfrac{1}{1-G/Y}\left(\frac{Y_t - Y}{Y} - \frac{G}{Y}\frac{G_t - G}{G}\right) \\
&\quad -\tfrac{1}{2}\tfrac{1}{(1-G/Y)^2}\left(\left(\frac{Y_t - Y}{Y}\right)^2 + \left(\frac{G}{Y}\right)^2\left(\frac{G_t - G}{G}\right)^2\right. \\
&\quad \left. -2\frac{YG}{Y^2}\left(\frac{Y_t - Y}{Y}\right)\left(\frac{G_t - G}{G}\right)\right) + o(\hat{y}_t^3) \\
&= \tfrac{1}{1-\gamma_G}(\hat{y}_t - \gamma_G \hat{g}_t) + \tfrac{1}{2}\tfrac{1}{1-\gamma_G}(\hat{y}_t^2 - \gamma_G \hat{g}_t^2) \\
&\quad -\tfrac{1}{2}\tfrac{1}{(1-\gamma_G)^2}(\hat{y}_t^2 + \gamma_G^2 \hat{g}_t^2 - 2\gamma_G \hat{y}_t \hat{g}_t) + o(\hat{y}_t^3) \\
&= \tfrac{1}{1-\gamma_G}(\hat{y}_t - \gamma_G \hat{g}_t) - \tfrac{1}{2}\tfrac{1}{(1-\gamma_G)^2}\left(\gamma_G\left((\tilde{g}_t - \tilde{y}_t) - (\hat{g}_t^n - \hat{y}_t^n)\right)^2\right) + o(\hat{y}_t^3) + t.i.p.
\end{aligned}$$

To a first-order approximation $\tilde{g}_t = \tilde{s}_t = \tilde{y}_t$, from A.4. The quadratic term therefore becomes independent of policy, $(\hat{g}_t^n - \hat{y}_t^n)^2$, because any second-order or higher terms in $(\tilde{g}_t - \tilde{y}_t)$ will become third-order or higher when squared. Substituting this second-order approximation into the goods market equilibrium in A.2 gives

$$\begin{aligned}
\ln C_t &= \ln(Y_t - G_t) - \alpha s_t - \ln[(1-\alpha) + \alpha\Theta_t^{-1} + (1-\alpha)\Theta_{J,t}\Theta_t^{-1}] \\
\tilde{c}_t &\approx \tfrac{1}{1-\gamma_G}(\tilde{y}_t - \gamma_G \tilde{g}_t) - \alpha \tilde{s}_t + o(\hat{y}_t^3) + t.i.p. \qquad \text{(B.1)}
\end{aligned}$$

Next we take a second-order log-linearisation of the third term in the loss function 2.1,

$$\begin{aligned}
\frac{N_t^{1+\varphi}}{1+\varphi} &= \frac{N^{1+\varphi}}{1+\varphi} + N^\varphi N\left(\frac{N_t - N}{N}\right) + \tfrac{1}{2}\varphi N^{\varphi-1}N^2\left(\frac{N_t - N}{N^2}\right)^2 + o(\hat{y}_t^3) \\
&= \frac{N^{1+\varphi}}{1+\varphi} + N^{1+\varphi}(\hat{n}_t + \tfrac{1}{2}\hat{n}_t^2) + \tfrac{1}{2}\varphi N^{1+\varphi}\hat{n}_t^2 + o(\hat{y}_t^3) \\
&= \hat{n}_t + \tfrac{1}{2}(1+\varphi)\hat{n}_t^2 + o(\hat{y}_t^3) + t.i.p. \qquad \text{(B.2)}
\end{aligned}$$

Where we assume that $N = 1$ in the steady state from Section A.4.2, and $\hat{n}_t = \tilde{n}_t + \hat{n}_t^n$. To express this in terms of the output gap we use $N_t = \left(\frac{Y_t}{A}\right) \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj$ and $N_t^n = \frac{Y_t^n}{A}$. Thus, we have $\tilde{n}_t = \tilde{y}_t + z_t$ where $z_t \equiv \ln \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj$, and $\hat{n}_t^n = \hat{y}_t^n$. Lemma 2 states that z_t is proportional to the cross-sectional variance of relative prices and thus of second order, so $\hat{n}_t^2 = (\tilde{y}_t + \hat{y}_t^n)^2 + o(\hat{y}_t^3)$. Lemma 3 states that the sum of the variance of relative prices can be expressed in terms of domestic inflation.

Lemma 2. $z_t = \frac{\epsilon}{2} var\{p_{H,t}(j)\} + o(\|\bar{g}_t\|^3)$

Proof. See Gali and Monacelli (2008), Appendix. □

Lemma 3. $\sum_{t=0}^{\infty} \beta^t var\{p_{H,t}(j)\} = \frac{1}{\lambda} \sum_{t=0}^{\infty} \beta^t \pi_{H,t}^2$ *where* $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$

Proof. See Woodford (2001), pp22-23. □

Now, in order to express welfare in terms of the output gap and inflation we will make use of the following log-linear equations, each of which is accurate to at least the second order: $\tilde{c}_t = (1-\alpha)\tilde{s}_t$; $\widetilde{mc}_t = \tilde{c}_t + \alpha\tilde{s}_t + \varphi\tilde{n}_t$; $\tilde{n}_t = \tilde{y}_t + z_t$; $\tilde{c}_t = \frac{1}{1-\gamma_G}(\tilde{y}_t - \gamma_G \tilde{g}_t) - \alpha\tilde{s}_t$; $\tilde{g}_t = \tilde{s}_t = \tilde{y}_t$. These follow from equations 2.11; 2.3; appendix A.3; B.1; $\hat{g}_t = \hat{s}_t + \widehat{rb}_t$ and 2.15, respectively. Combining the last three gives $\tilde{n}_t = (1-\gamma_G)(\tilde{c}_t + \alpha\tilde{s}_t) + \gamma_G\tilde{s}_t + z_t$. Combining all with 2.1 and B.2 gives the loss function,

$$\mathbb{L}_0 = E_0 \sum_t^{\infty} \beta^t \left\{ \tfrac{1}{2}\tfrac{\epsilon}{\lambda}\pi_{H,t}^2 + \tfrac{1}{2}(1+\varphi)(\tilde{y}_t + \hat{y}_t^n)^2 + \alpha\,(1-\chi)\,\tilde{s}_t \right\} + o(\hat{y}_t^3) + t.i.p.$$

This takes the familiar linear-quadratic form, with the exception of the linear $\tilde{s}_t$term. This linear term can alter the welfare ranking of policies unless the fluctuations in $\tilde{s}_t$ are “small” (of second-order or above, see Benigno and Woodford, 2005). To remedy this we re-express the loss function using only quadratic terms, making use of the second-order approximation of aggregate supply in A.13. First we express the loss function in terms of marginal costs. The terms of trade gap can be expressed as $\tilde{s}_t = \frac{1}{1+\varphi}(\widetilde{mc}_t - \varphi z_t)$. Substituting this into the loss function and using Lemmas 2 and 3 gives, $\mathbb{L}_0 = E_0 \sum_t^{\infty} \beta^t \left\{ \frac{1}{2}\frac{\epsilon}{\lambda}(1 - \frac{\alpha\varphi(1-\chi)}{1+\varphi})\pi_{H,t}^2 + \frac{1}{2}(1+\varphi)(\tilde{y}_t + \hat{y}_t^n)^2 + \frac{\alpha(1-\chi)}{1+\varphi}\widetilde{mc}_t \right\} + o(\hat{y}_t^3) + t.i.p.$. Now multiplying A.13 by $\frac{\alpha(1-\chi)}{(1+\varphi)\lambda}$ and substituting into this loss function again gives, $\mathbb{L}_0 = E_0 \sum_t^{\infty} \beta^t \left\{ \frac{1}{2}\phi_\pi \pi_{H,t}^2 + \frac{1}{2}\phi_y \tilde{y}_t^2 + (1+\varphi)\tilde{y}_t\hat{y}_t^n \right\} - T_0 + o(\hat{y}_t^3) + t.i.p.$, where $\phi_\pi = \frac{\epsilon}{\lambda}(1 - \frac{\alpha\varphi(1-\chi)}{1+\varphi}) + \frac{\alpha(1-\chi)}{(1+\varphi)}c_\pi$, $\phi_y = (1+\varphi)(1+\alpha(1-\chi))$ and $T_0 = \frac{\alpha(1-\chi)}{(1+\varphi)\lambda}V_0$ is a transitory component following Benigno and Woodford (2005). T_0 is predetermined and thus independent of monetary policy from a timeless perspective. This approach precludes using the loss function for studying discretionary policy (Benigno and Woodford, 2005). The loss function only consists of terms of second or higher order, and so will be accurate to the first order when differentiated, including for larger deviations of the natural level of output from its original level. Therefore, we are able to rank policies using the loss function in equation 3.1.

B.2 Optimal Policy from a Timeless Perspective

Optimal policy from a timeless perspective (treating T_0 as transitory) can be found by solving the Lagrangian,

$$\begin{aligned}\min_{\pi_{Ht},\tilde{y}_t} \quad \mathcal{L}_{\mathcal{TL}} = \quad & E_0 \sum_t^{\infty} \beta^t \left\{ \tfrac{1}{2}\phi_\pi \pi_{H,t}^2 + \tfrac{1}{2}\phi_y \tilde{y}_t^2 + (1+\varphi)\tilde{y}_t \hat{y}_t^n \right\} \\ & -l_t(\beta\pi_{H,t+1} + \lambda(1+\varphi)\tilde{y}_t - \pi_{H,t}) \end{aligned} \tag{B.3}$$

where $\phi_\pi = \frac{\epsilon}{\lambda}(1 - \frac{\alpha\varphi(1-\chi)}{1+\varphi}) + \frac{\alpha(1-\chi)}{(1+\varphi)}c_\pi$ and $\phi_y = (1+\varphi)\,(1+\alpha(1-\chi))$. This gives the FOCs $\phi_\pi\pi_{H,t} + l_t = l_{t-1}$, $\phi_y\tilde{y}_t + (1+\varphi)\hat{y}_t^n = \lambda(1+\varphi)l_t$ and $\beta\pi_{H,t+1} + \lambda(1+\varphi)\tilde{y}_t = \pi_{H,t}$. Combining the first two gives $\tilde{y}_0 + \frac{(1+\varphi)}{\phi_y}\hat{y}_0^n = -\frac{\lambda(1+\varphi)\phi_\pi}{\phi_y}\pi_{H,0}$ and $\tilde{y}_t + \frac{(1+\varphi)}{\phi_y}\hat{y}_t^n = \tilde{y}_{t-1} + \frac{(1+\varphi)}{\phi_y}\hat{y}_{t-1}^n - \frac{\lambda(1+\varphi)\phi_\pi}{\phi_y}\pi_{H,t}$, with the Phillips curve in period -1 not being a binding constraint in period 0. Iteratively combining these gives the following, where $\hat{p}_{H,t} = p_{H,t} - p_{H,-1}$,

$$\tilde{y}_t \quad = \quad -\frac{\lambda(1+\varphi)\phi_\pi}{\phi_y}\hat{p}_{H,t} - \frac{(1+\varphi)}{\phi_y}\hat{y}_t^n \tag{B.4}$$

Substituting this into the Phillips curve gives $\hat{p}_{H,t} - \hat{p}_{H,t-1} = \beta E_t[\hat{p}_{H,t+1}] - \beta\hat{p}_{H,t} + \lambda(1+\varphi)\left(-\frac{\lambda(1+\varphi)\phi_\pi}{\phi_y}\hat{p}_{H,t} - \frac{(1+\varphi)}{\phi_y}\hat{y}_t^n\right)$ and $E_t[\hat{p}_{H,t+1}] = (a\beta)^{-1}\hat{p}_{H,t} - \beta^{-1}\hat{p}_{H,t-1} + b\beta^{-1}\hat{y}_t^n$, where $a = \left(1+\beta+\lambda^2(1+\varphi)^2\frac{\phi_\pi}{\phi_y}\right)^{-1}$ and $b = \frac{\lambda(1+\varphi)^2}{\phi_y}$. To find the stationary solution to this price process we rearrange and make use of the lag operator L to give, $\hat{p}_{H,t}E_t[L(\frac{1}{\beta} - \frac{1}{a\beta}L^{-1} + L^{-2})] = b\beta^{-1}\hat{y}_t^n$, where $\hat{p}_{H,t-1} = L\hat{p}_{H,t}$. We now factorize the quadratic expression $(\frac{1}{\beta} - \frac{1}{a\beta}L^{-1} + L^{-2}) = (L^{-1} - \delta_1)(L^{-1} - \delta_2)$. To do so let $z = L^{-1}$, so that $(\frac{1}{\beta} - \frac{1}{a\beta}z + z^2) = (z - \delta_1)(z - \delta_2)$, and $\delta_i = \left(1 \pm \sqrt{1-4a^2\beta}\right)/2a\beta$ for $i = 1, 2$. We assume that $\mid \delta_1 \mid < 1$ and $\mid \delta_2 \mid > 1$. Substituting this factorization into the above expression yields, $\hat{p}_{H,t}(1 - \delta_1 L) = E_t\left[(b\beta^{-1}\hat{y}_t^n)\,(L^{-1} - \delta_2)^{-1}\right]$. Now, we can express the term $(L^{-1} - \delta_2)^{-1}$ as an infinite geometric series where the coefficients converge to zero because $\mid \delta_2 \mid > 1$, $(L^{-1} - \delta_2)^{-1} = -\delta_2^{-1}\left(1 + L^{-1}\delta_2^{-1} + L^{-2}\delta_2^{-2} + \ldots\right)$. Substituting this into the above expression gives, $\hat{p}_{H,t}(1 - \delta_1 L) = -(\beta\delta_2)^{-1}bE_t\left[\sum_{s=0}^{\infty}(\delta_2^{-s}\hat{y}_{t+s}^n)\right]$. Finally, multiplying the numerator and denominator of each term with $(1 - \sqrt{1-4a^2\beta})$, and simplifying notation $\delta_1 = \delta = (1 - \sqrt{1 - 4a^2\beta})/(2a\beta)$ gives the stationary solution below,

$$\hat{p}_{H,t} = \delta \hat{p}_{H,t-1} - b\delta E_t \left[\sum_{s=0}^{\infty} (\beta\delta)^s \hat{y}^n_{t+s} \right] \tag{B.5}$$

An interest rate rule that brings about a unique, determinate equilibrium can be found by combining B.4, B.5 and the IS curve, to give the expression in 3.4.

B.3 Appendix: Exchange Rate Peg

Here we derive the dynamics of the economy under a nominal exchange rate peg, $\Delta e_t = 0$. We begin by finding the implications of the nominal exchange rate peg for our key variables, $\pi_{H,t}$ and $\hat{y}_t$. To do this we follow a similar approach to Appendix B.2.

First, taking first differences of the effective nominal exchange rate yields $\Delta s_t = \Delta e_t + \Delta p^*_t - \Delta p_{H,t} = -\pi_{H,t}$. Also, taking first differences of A.4 gives $\Delta \hat{s}_t = \Delta \hat{y}_t - \gamma_G \Delta \widehat{rb}_t - (1 - \gamma_G)(\Delta \hat{c}^*_t + (1 - \alpha)(\Delta \hat{\vartheta}_t + \Delta \hat{\vartheta}_{J,t}))$. Combining these two expressions gives, $\pi_{H,t} = \gamma_G \Delta \widehat{rb}_t - \Delta \hat{y}_t + (1 - \gamma_G)(\Delta \hat{c}^*_t + (1 - \alpha)(\Delta \hat{\vartheta}_t + \Delta \hat{\vartheta}_{J,t}))$. Taking a similar approach to that used in the optimal policy case we evaluate this expression at time zero, assuming that the economy was in the steady state at all times prior to this, to give $p_{H,0} - p_{H,0-1} = \gamma_G \widehat{rb}_0 - \hat{y}_0 + (1 - \gamma_G)(\hat{c}^*_0 + (1 - \alpha)(\hat{\vartheta}_0 + \Delta \hat{\vartheta}_{J,0}))$. Iteratively combining these two relationships allows us to express this relationship in terms of the deviation of the domestic price level, $\hat{p}_{H,t} = \gamma_G \widehat{rb}_t - \hat{y}_t + (1 - \gamma_G)\left(\hat{c}^*_t + (1 - \alpha)(\hat{\vartheta}_t + \hat{\vartheta}_{J,t})\right)$ where $\hat{p}_{H,t} = p_{H,t} - p_{H,-1}$. This can also be written as $\hat{p}_{H,t} = -\hat{y}_t + (1 - \gamma_G)\hat{y}^*_t + \hat{\vartheta}_t + (1 + \varphi)(\hat{y}^n_t - \hat{a}_t)$. Using this to substitute out $\hat{y}_t$ in the Phillips curve gives, $\hat{p}_{H,t} - c\hat{p}_{H,t-1} - c\beta E_t[\hat{p}_{H,t+1}] = c\lambda(1 + \varphi)(\varphi \hat{y}^n_t + \hat{\vartheta}_t + (1 - \gamma_G)\hat{y}^*_t - (1 + \varphi)\hat{a}_t)$, where $c = [1 + \beta + \lambda(1 + \varphi)]^{-1}$. We can find the closed-form stationary solution to this linear difference equation following the method described in Appendix B.2, $\hat{p}_{H,t} = \delta_c \hat{p}_{H,t-1} + \lambda(1 + \varphi)\left\{\delta_c E_t \left[\sum_{s=0}^{\infty} (\beta\delta_c)^s \left(\varphi \hat{y}^n_{t+s} + (1 - \gamma_G)\hat{y}^*_{t+s} - (1 + \varphi)\hat{a}_{t+s}\right)\right] + \frac{\delta_c}{1 - \beta\delta_c}\hat{\vartheta}_t\right\}$, where $\delta_c = \frac{1 - \sqrt{1 - 4\beta c^2}}{2\beta c}$. We can use this to derive the output gap using the relationship defined above, and for the nominal interest rate by substituting our results into the IS curve. By uncovered interest parity the nominal interest rate will be constant. Uncovered interest parity states that, $(i_t - i^*_t) - E_t[\pi_{H,t+1}] = E_t[s_{t+1} - s_t]$. The definition of the terms of trade yields, $s_t = e_t + p^*_t - p_{H,t}$. So, $(i_t - i^*_t) = E_t[e_{t+1} - e_t] = 0$. While the nominal interest rate stays constant there will be relatively large fluctuations in the real interest rate. Note that while an exchange rate peg is consistent with a constant nominal interest rate, it is not uniquely associated with it. A commitment to the peg remains crucial.

B.4 Appendix: Taylor Rules

Here we derive the dynamics of the economy under a variety of Taylor rules. To do this we extend the Blanchard and Kahn (1980) method to allow for anticipated changes in oil revenue. As in Blanchard and Kahn this uses the widely implementable eigen-decomposition of a matrix. The method is similar to that used by Wohltmann and Winkler (2009), who use a generalized Schur matrix decomposition. As we will be interested in interest rate rules that respond to the output gap, we arrange the Philips curve and IS curve into a system of two equations in domestic inflation and the output gap,

$$\begin{aligned} E_t[\pi_{H,t+1}] &= \beta^{-1}\left(\pi_{H,t} - \lambda(1+\varphi)\tilde{y}_t\right) \\ E_t[\tilde{y}_{t+1}] &= \tilde{y}_t + (i_t - \rho) - \beta^{-1}\left(\pi_{H,t} - \lambda(1+\varphi)\tilde{y}_t\right) - (1+\varphi)E_t[\Delta a_{t+1}] + \varphi E_t[\Delta \hat{y}^n_{t+1}] \end{aligned}$$

where $(i_t - \rho) = \theta_\pi \pi_{H,t} + \theta_y \tilde{y}_t + \theta_{ya} E_t[\Delta a_{t+1}] + \theta_{yn} E_t[\Delta \hat{y}^n_{t+1}]$. This can be arranged in matrix notation, splitting the variables into control variables, $x_t = [\pi_{H,t}, \tilde{y}_t]'$, and state variables $w_t = \emptyset$ and $v_t = [\hat{y}^n_t, \hat{y}^n_{t+1}, \hat{a}_t, \hat{a}_{t+1}]'$, to give $E_t \begin{bmatrix} x_{t+1} \\ w_{t+1} \end{bmatrix} = A \begin{bmatrix} x_t \\ w_t \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} v_t$. Decomposing A into its eigenvectors, V, and eigenvalues, Λ, gives $AV = V\Lambda$. The matrix Λ is diagonal with the eigenvalues arranged in descending order along the diagonal. Replacing $A = V\Lambda V^{-1}$ and pre-multiplying by V^{-1} gives, $V^{-1}E_t \begin{bmatrix} x_{t+1} \\ w_{t+1} \end{bmatrix} = \Lambda V^{-1} \begin{bmatrix} x_t \\ w_t \end{bmatrix} + V^{-1} \begin{bmatrix} B \\ 0 \end{bmatrix} v_t$. Now let $V^{-1} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$ and $\Lambda = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix}$. For this work, where the state variables evolve independently of the controls, $Y_{21} = 0$. The matrix Λ_1 contains unstable eigenvalues (> 1) which are equal in number to the control variables, and Λ_2 contains stable eigenvalues (< 1) which are equal in number to the state variables, as imposed by Blanchard and Kahn (1980). Using $\begin{bmatrix} \tilde{x}_t \\ \tilde{w}_t \end{bmatrix} = V^{-1} \begin{bmatrix} x_t \\ w_t \end{bmatrix}$ we can describe the system in independent equations, $E_t \begin{bmatrix} \tilde{x}_{t+1} \\ \tilde{w}_{t+1} \end{bmatrix} = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \begin{bmatrix} \tilde{x}_t \\ \tilde{w}_t \end{bmatrix} + \begin{bmatrix} \tilde{B}_1 \\ \tilde{B}_2 \end{bmatrix} v_t$, where $\tilde{B}_1 = Y_{11}B$ and $\tilde{B}_2 = Y_{21}B$. First taking $E_t[\tilde{x}_{t+1}] = \Lambda_1 \tilde{x}_t + \tilde{B}_1 v_t$. This can be expressed as, $(E_t[L^{-1}] - \Lambda_1)\tilde{x}_t = \tilde{B}_1 v_t$ where L^{-1} is a scalar inverse lag operator. Rearranging gives $\tilde{x}_t = (E_t[L^{-1}] - \Lambda_1)^{-1}\tilde{B}_1 v_t = -\Lambda_1^{-1}(I - E_t[L^{-1}]\Lambda_1^{-1})^{-1}\tilde{B}_1 v_t$. We can only accept stable paths for the control variables. As all the elements of $\Lambda_1$are greater than one, the eigenvalues of Λ_1^{-1} will be less than one and the matrix geometric series, $(I -$

$E_t[L^{-1}]\Lambda_1^{-1})^{-1} = \sum_{s=0}^{\infty}(E_t[L^{-1}]\Lambda_1^{-1})^s$ will converge. Thus, $\tilde{x}_t = -\Lambda_1^{-1}\sum_{s=0}^{\infty}(\Lambda_1^{-1})^s\tilde{B}_1E_t[v_{t+s}]$ and $x_t = -(Y_{11})^{-1}Y_{12}w_t - (Y_{11})^{-1}\Lambda_1^{-1}\sum_{s=0}^{\infty}(\Lambda_1^{-1})^s\tilde{B}_1E_t[v_{t+s}]$.

Turning now to the path of the state variables, $E_t[\tilde{w}_{t+1}] = \Lambda_2\tilde{w}_t + \tilde{B}_2v_t$. We have $\tilde{w}_t = Y_{21}x_t + Y_{22}w_t = Kw_t - J\sum_{s=0}^{\infty}(\Lambda_1^{-1})^s\tilde{B}_1E_t[v_{t+s}]$, where $K = -Y_{21}(Y_{11})^{-1}Y_{12} + Y_{22}$ and $J = Y_{21}(Y_{11})^{-1}\Lambda_1^{-1}$. Therefore, the path of the state variables can be described as, $E_t[w_{t+1}] = K^{-1}\Big(\Lambda_2Kw_t - \Lambda_2J\sum_{s=0}^{\infty}(\Lambda_1^{-1})^s\tilde{B}_1E_t[v_{t+s}] + J\sum_{s=0}^{\infty}(\Lambda_1^{-1})^s\tilde{B}_1E_t[v_{t+1+s}] + \tilde{B}_2v_t\Big)$.

C Appendix: Calibration

The model is calibrated to a typical small open economy like Canada following Gali and Monacelli (2005, 2008), to make the results comparable. The calibrated parameter values are: $\alpha = 0.4$, $\beta = 0.99$, $\rho = 0.01$, $\chi = 0.2$, $\varphi = 3$, $\epsilon = 6$, $\theta = 0.75$, $A = A^* = 1$, $\gamma_G = 0.25$, $\gamma_T = 0.2$, $\gamma_J = 0.05$. The steady state values are (symmetric steady state in parentheses): $N = 1(1)$, $G = 0.25(0.25)$, $T = 0.2(0.25)$, $O = 0.73(0)$, $P_O = 1(1)$, $\Theta = 1.46(1)$, $S = 0.69(1)$, $C = 0.93(0.75)$, $J = 0.035(0)$.

D Online Appendix: Robustness

D.1 Approximating relative household wealth

To make the analysis tractable enough to permit a closed form solution I approximate relative household wealth as $-\alpha\hat{\vartheta}_t \approx \ln(1-\alpha+\alpha\Theta_t^{-1}) - \ln(1-\alpha+\alpha\Theta_0^{-1})$ when log-linearising the goods market equilibrium (equation A.2). I test this for accuracy against three different approximations, each implemented in Dynare. The first uses a Newton-Raphson algorithm applied directly to the model in levels ("*NR*"). The second takes a first-order log-linearisation of the model around the initial steady state, but keeps track of both $\hat{\theta}_t = \ln(1-\alpha+\alpha\Theta_t^{-1}) - \ln(1-\alpha+\alpha\Theta_0^{-1})$ and $\hat{\vartheta}_t = \ln(\Theta_t) - \ln(\Theta_0)$ ("$\theta + \vartheta$"). The third again takes a first-order log-linearisation of the model, but approximates $-\alpha\hat{\vartheta}_t \approx \hat{\theta}_t$ as reported in the main text ("ϑ"). Prices are assumed to be flexible in each. As illustrated in Figure D.1, these approximations make little material difference to our analysis.

D.2 Lump sum taxes

In log-linearising the government spending rule, $\hat{g}_t = \hat{s}_t - \hat{p}_t^* + \widehat{rb}_t - \hat{t}_t$, we make a simplifying assumption regarding lump sum taxes. We define $t_t = -\ln(1 - \frac{T_t}{G_t})$, so that the log-linearized government spending rule is exact. We also assume that $\hat{t}_t = 0$, so the ratio of lump sum taxes

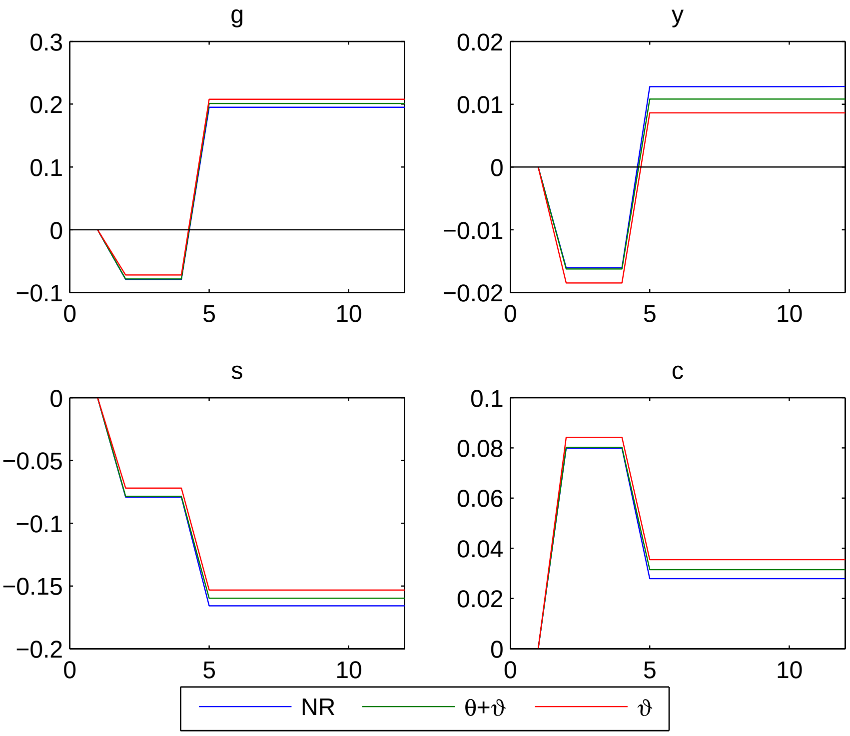

Figure D.1: Robustness to approximating ϑ numerically using the Newton-Raphson method ("NR"), log-linearized making a distinction between θ and ϑ ("$\theta+\vartheta$"), and log-linearized using the approximation, $\theta \approx -\alpha\vartheta$ ("ϑ"; used in text).

to government spending remains constant. Alternatively, we could have set the level of taxes to be constant. As illustrated in the simulations in Figure D.2, this assumption changes the magnitude of the government spending shock but the qualitative results remain the same.

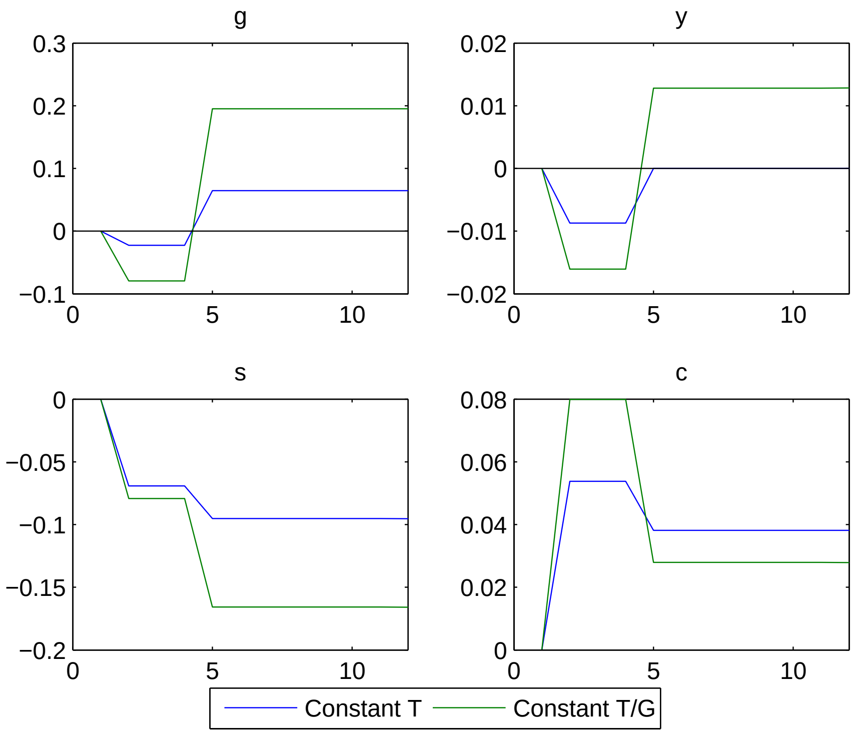

Figure D.2: Comparing different tax assumptions: constant T and constant T/G.