MSc in Economics for Development

Macroeconomics for Development

Week 6 Class

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Consultation hours: Friday, 2-3pm, Weeks 1,3-8 (MT)

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References:

- Romer, D., 2001, *Advanced Macroeconomics*, McGraw-Hill Higher Education
  - Chapter 2, especially 2.3, 2.6, 2.7
  - Focus on phase diagrams rather than log-linearisation
- Heijdra, B. J., Van der Ploeg, F., 2002, *Foundations of Modern Macroeconomics,* OUP, Ch 14.5
  - Alternative treatment
- Macro Class 2:
  - Phase Diagrams and Dynamic Systems

Overview: the Ramsey model

- The Ramsey model's key characteristic is an endogenous savings rate
- It can be summarised in 2 dynamic equations and represented in a phase diagram
- The dynamics of c are derived from optimising household behaviour, and those of k are defined
- The phase diagram can be constructed using the stationary locii of c and k
- At the steady state the economy is on the balanced growth path, though consumption is not maximised in perpetuity
- Away from the steady state, consumption is chosen to move to the stable saddle path
- If we start at the steady state, then we can analyse the dynamics associated with shocks to the economy
    1. Fall in the discount rate
    2. Permanent increase in Government spending
    3. Temporary increase in Government spending

FROM CLASS 2: "The Solow-Swan model was discussed in the lecture. There have been a wide range of extensions to it."

Solow-Swan Limitation	Extension

1. Constant savings rate	• Endogenise savings/consumption (Ramsey, 1928): This Class
	  • Using technological change (Lucas, 1988)
	  • Using capital accumulation (Jones and Manuelli, 1990)

2. Omitted factors esp. human capital ("Lucas Paradox", Lucas, AER, 1990)	• Include human capital:
	  • As schooling (Mankiw, Romer, Weil)
	  • As knowledge (Romer, 1986)
	  • Y=AK model (Romer, 1986)
	  • As skill (Lucas, 1988)
	  • As learning by doing (Young, 1999)

3. Exogenous technical change	•Endogenise technical change:
	  • Using R&D (Romer, 1990) Class 2
	  • Using Schumpeterian competition (Grossman and Helpman, 1991)
	  • Using both R&D and Schumpeterian competition (Aghion and Howitt, 1992)

4. Aggregation production function assumes optimal resource allocation across economy	• Develop non-aggregate theory (Solow, 2005)

Summary of the Ramsey-Cass-Koopmans model

The Ramsey model's key characteristic is an endogenous savings rate

- Endogenous savings rate
- Perfectly competitive firms rent capital and employ labour owned by households (GE)
  - No market imperfections
- Households are infinitely lived

The Ramsey Model can be summarised in two dynamic equations

Dynamics of Consumption (c)

$$\frac{\dot{c}(t)}{c(t)} = \frac{f'(k(t)) - \rho - \theta g}{\theta}$$

ρ discount rate
ϑ 1/IES
g tech. growth

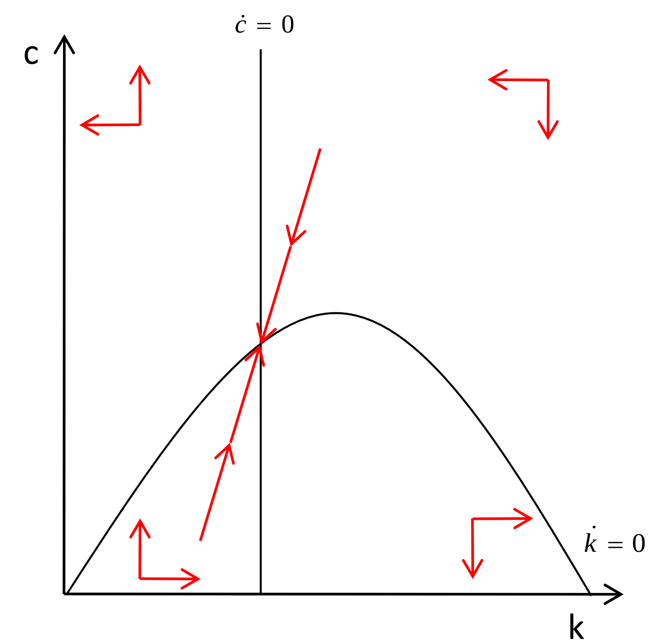
- Derived from maximising consumer (choice variable)

Dynamics of Capital (k)

$$\dot{k}(t) = f(k(t)) - c(t) - (n+g)k(t)$$

- Defined as the difference between actual investment and break-even investment (state variable)

...which can be illustrated in a phase diagram



To derive the dynamics of consumption we consider how households optimally choose their level of c

Household behaviour is summarised by maximising the Lagrangian

$$\max_c \; \mathfrak{L} = \overbrace{B\int_{t=0}^{\infty} e^{-\beta t}\frac{c(t)^{1-\theta}}{1-\theta}dt}^{\text{max utility}} + \lambda\overbrace{\left[k(0) + \int_{t=0}^{\infty} e^{-R(t)}e^{(n+g)t}w(t)dt - \int_{t=0}^{\infty} e^{-R(t)}e^{(n+g)t}c(t)dt\right]}^{\text{subject to budget constraint (lifetime wages > consump.)}}$$

see Romer eqn 2.16

- B adjustment for units of effective labour
  $B = A(0)^{1-\theta}L(0)/H$
- β discount factor:
  $\beta = \rho - n - (1-\theta)g > 0$
- ρ discount rate
- n population growth
- θ 1/IES (intertemporal elasticity of substitution)
- g technology growth
- $R(t)$ real interest rate between time 0 and t
- w wages per unit effective labour
- c consumption per unit effective labour
- $k(0)$ initial capital

This gives the first-order conditions

$$\frac{\partial \mathfrak{L}}{\partial c(t)} = Be^{-\beta t}c(t)^{-\theta} - \lambda e^{-R(t)}e^{(n+g)t} = 0$$

$$\frac{\partial \mathfrak{L}}{\partial \lambda} = \text{budget constraint}$$

Rearranging (i), taking logs (ii) and differentiating (iii) gives the equation of motion for consumption (the Euler Eqn)

i) $$Be^{-\beta t}c(t)^{-\theta} = \lambda e^{-R(t)}e^{(n+g)t}$$

ii) $$\ln B - \beta t - \theta \ln c(t) = \ln \lambda - R(t) + (n+g)t$$

iii) $$\beta - \theta\frac{\dot{c}(t)}{c(t)} = -r(t) + (n+g)$$

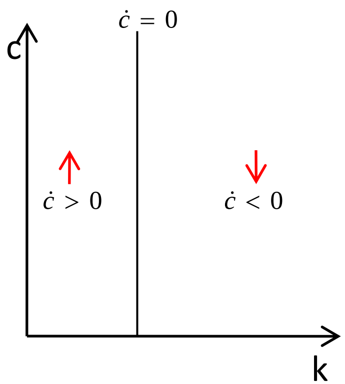
$$\frac{\dot{c}(t)}{c(t)} = \frac{r(t) - (n+g) - \beta}{\theta}$$

$$\frac{\dot{c}(t)}{c(t)} = \frac{r(t) - \rho - \theta g}{\theta}$$

Euler Equation

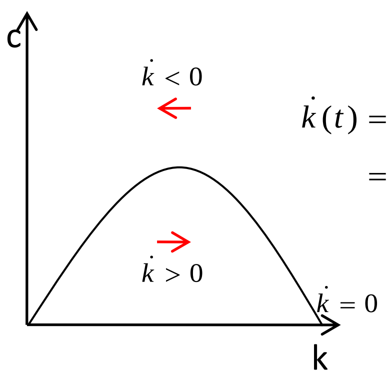
The phase diagram can be constructed with reference to the stationary locii for c and k

Stationary locus for c



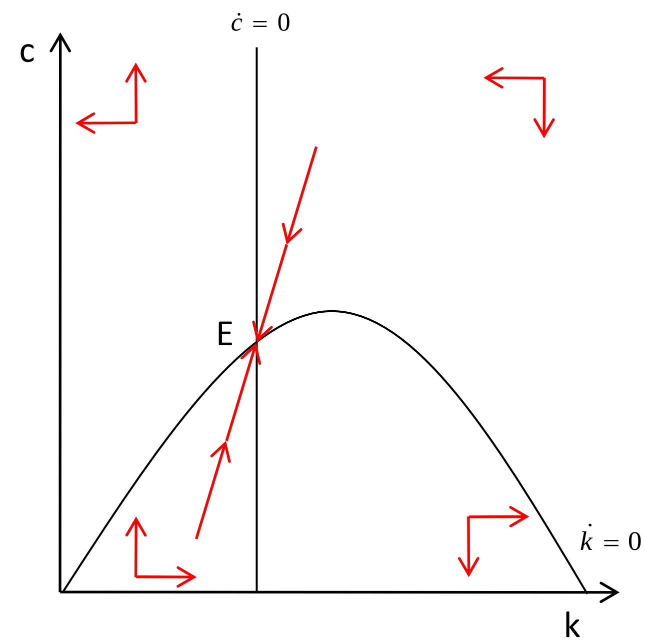
$$\frac{\dot{c}(t)}{c(t)} = \frac{f'(k(t)) - \rho - \theta g}{\theta}$$
$$= 0$$

Stationary locus for k



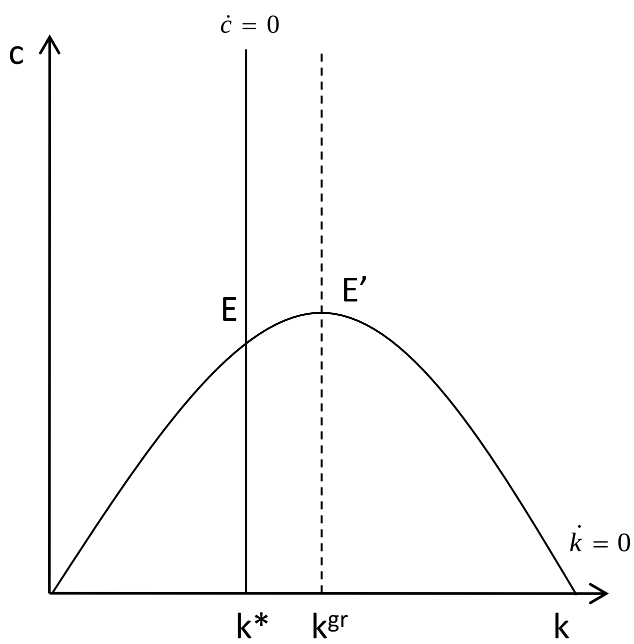
$$\dot{k}(t) = f(k(t)) - c(t) - (n + g)k(t)$$
$$= 0$$

Phase diagram for c and k



At the steady state the economy is on the balanced growth path, though consumption is not maximised in perpetuity

The steady state is at the intersection of the stationary locii



$$\dot{c}(t) = 0: \quad f'(k(t)) = \rho + \theta g$$

$$\dot{k}(t) = 0: \quad c(t) = f(k(t)) - (n+g)k(t)$$

The steady state levels of consumption and capital do not maximise consumption in perpetuity

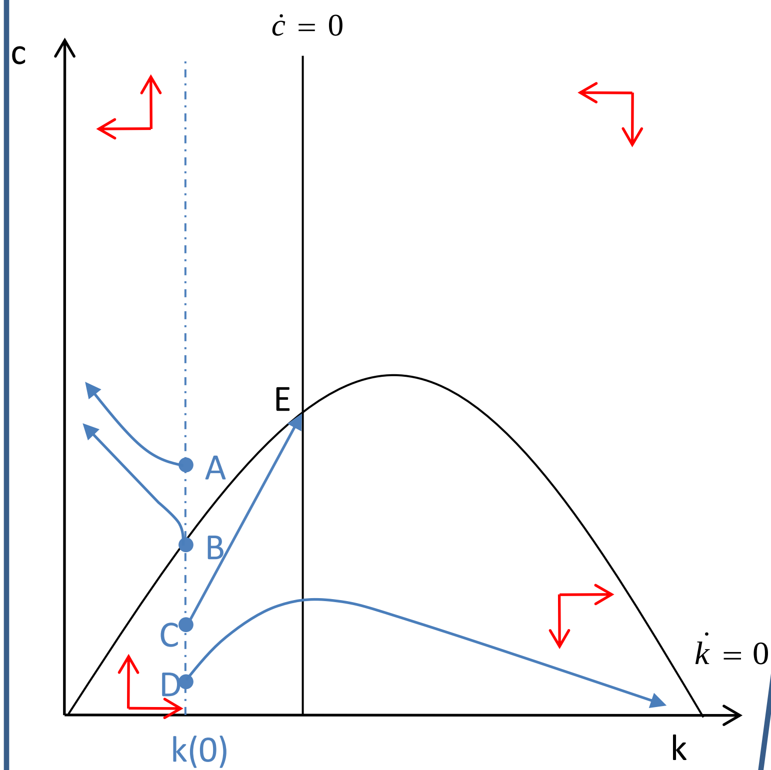
- On the k stationary locus, c is highest when $f'(k)=n+g$, known as the “golden rule” level of capital $k=k^{gr}$
- The steady state level of capital is below this, $k^*<k^{gr}$, as:
  - $f'(k^*)=\rho+\vartheta g$
  - $f'(k^{gr}) = n+g$

  and
  - $\beta = \rho - n - (1-\vartheta)g > 0$
    - This prevents lifetime utility from diverging as the contribution of technology growth to utility outweighs the reduction in utility due to preferences *(ρ)*and population growth (n)
- Intuitively:
  - E<E’ because consumers discount the future and so prefer to consume some more now, reducing future *k*.
  - E (not >) E’ as could consume more now and in future

- At the steady state E the savings rate is constant and C grows at rate n+g (as in Solow model). Technology still drives growth.

Away from the steady state, consumption is chosen to move to the stable saddle path

For a given level of capital, consumption is chosen to lie on the stable saddle path



Formally this is done by applying budget constraints to the path of consumption

- Given an initial value of capital k(0), the corresponding initial value of c must be chosen

•All options (A, B, C,D) satisfy at each point in time

$$\frac{\dot{c}(t)}{c(t)} = \frac{f'(k(t)) - \rho - \theta g}{\theta}$$

$$\dot{k}(t) = f(k(t)) - c(t) - (n+g)k(t)$$

•However, the consumer is also subject to the budget constraint:

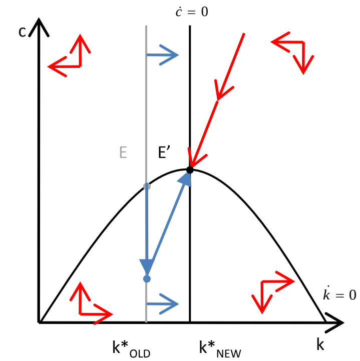
$$k(0) + \int_{t=0}^{\infty} e^{-R(t)} e^{(n+g)t} w(t) dt - \int_{t=0}^{\infty} e^{-R(t)} e^{(n+g)t} c(t) dt = 0$$

expressed as "no Ponzi game" transversality condition: $\lim_{s \to \infty} e^{-R(s)} e^{(n+g)s} k(s) = 0$

- Above C: consuming capital which eventually becomes negative
- Below C: k(s) rises and R(s) falls causing the transversality condition to diverge to infinity

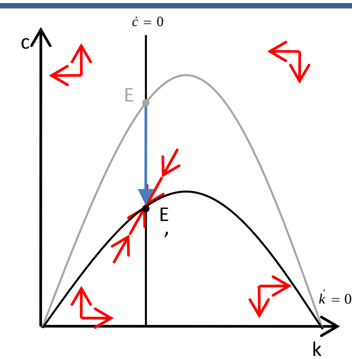
If we start at the steady state, then we can analyse the dynamics associated with shocks to the economy

1. Fall in the discount rate

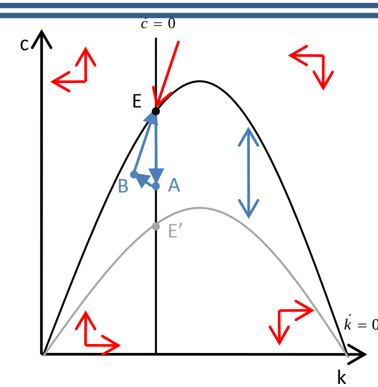


2. Increase in government spending

a. Permanent increase

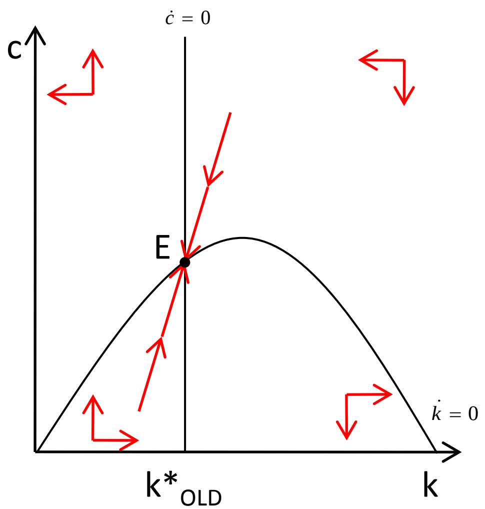


b. Temporary increase



Shocks to the economy:
1. Fall in the discount rate

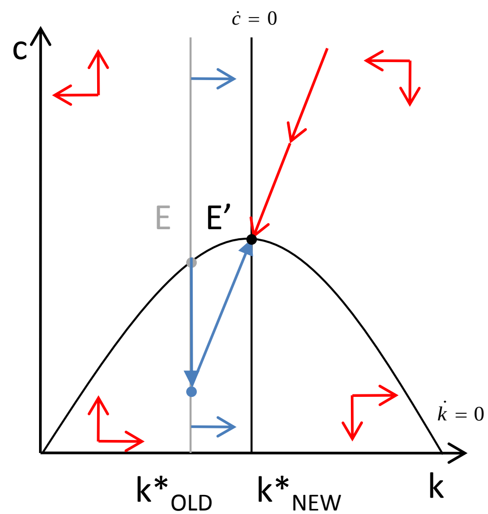
Before the shock the economy is at the original steady state

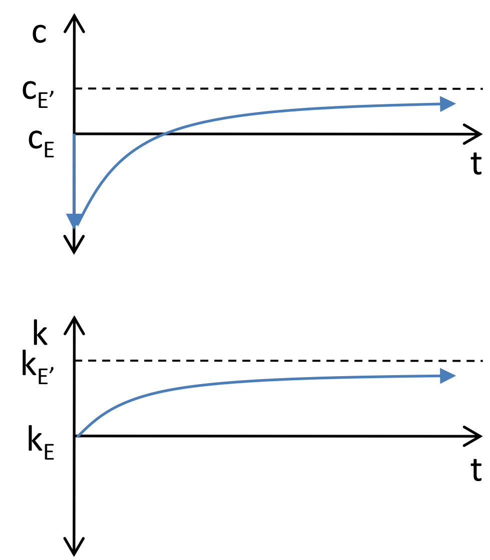


$$\frac{\dot{c}(t)}{c(t)} = \frac{f'(k(t)) - \rho - \theta g}{\theta}$$

$$\dot{k}(t) = f(k(t)) - c(t) - (n+g)k(t)$$

After the shock consumption jumps immediately on to the new saddle path

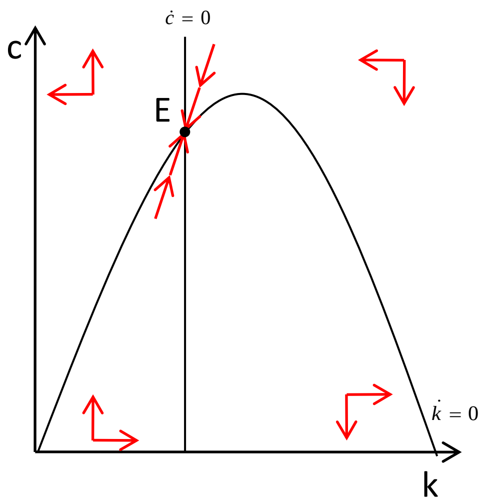




- Consumers care more about future consumption, so save more in the present

Shocks to the economy:
2a. Permanent increase in government spending

Before the shock the economy is at the original steady state

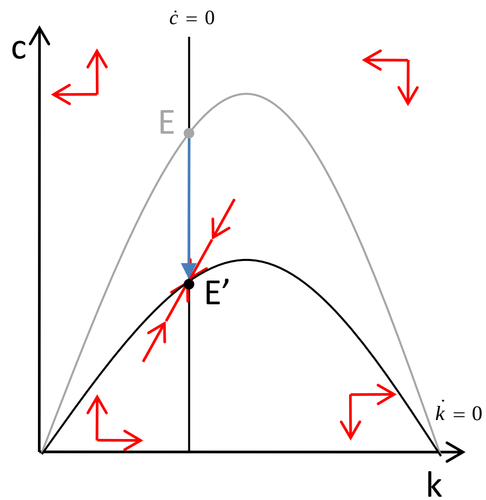


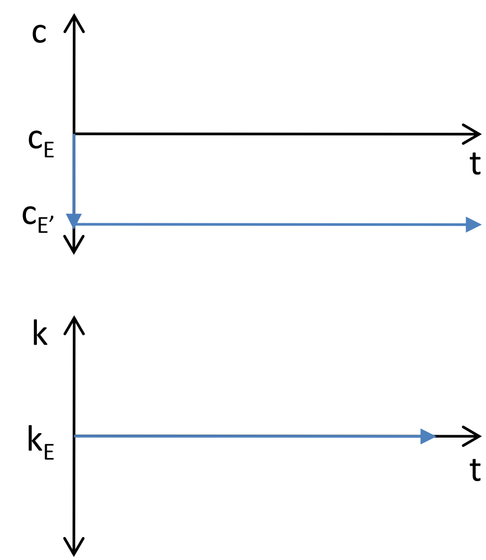
$$\frac{\dot{c}(t)}{c(t)} = \frac{f'(k(t)) - \rho - \theta g}{\theta}$$

adding gov't

$$\dot{k}(t) = f(k(t)) - c(t) - G(t) - (n+g)k(t)$$

After the shock consumption jumps immediately to the new level of equilibrium consumption

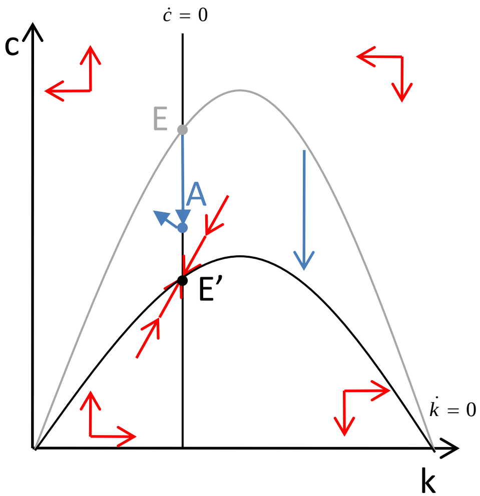




- Government consumption reduces the amount available to consumers if maintaining the same level of capital

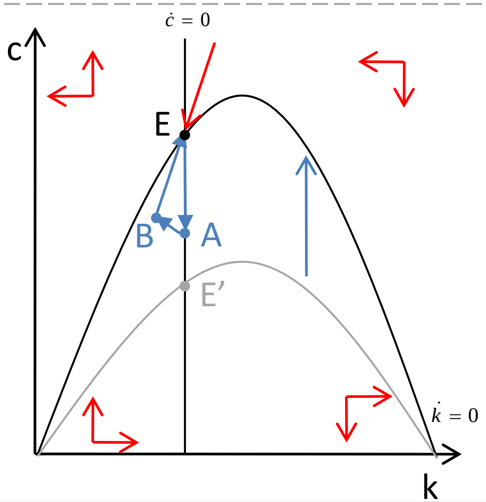
Shocks to the economy:
2b. Temporary increase in government spending

A temporary increase in government spending induces dynamics both during and after the shock



Whilst G is temporarily higher

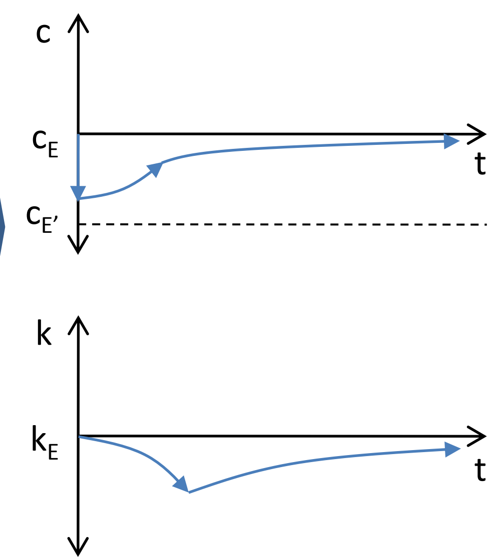
- Increase in government spending shifts the stationary k locus down for the duration of the shock
- Consumption adjusts immediately, but not all the way to the temporary equilibrium E' (as there is perfect foresight on when the shock ends)
- Capital starts to be consumed...



When G returns to original levels

- The consumer has consumed with perfect foresight the exact amount of capital needed to be on the saddle path when G falls again
- As the economy is on the stable saddle path it returns to the initial equilibrium

The fall in private c is smoothed by consuming capital



Summary: the Ramsey model

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